Spin-wave contributions to current-induced domain wall dynamics

Yann Le Maho* and Joo-Von Kim†
Institut d'Electronique Fondamentale, CNRS, UMR 8622, 91405 Orsay, France
and Université Paris Sud, UMR 8622, 91405 Orsay, France

Gen Tatara‡
Graduate School of Science, Tokyo Metropolitan University, 1-1 Minamiosawa, Hachioji, Tokyo 192-0397, Japan
(Received 22 December 2008; published 4 May 2009)

We examine theoretically the role of spin waves on current-induced domain wall dynamics in a ferromagnetic wire. At room temperature, we find that an interaction between the domain wall and the spin waves appears when there is a finite difference between the domain wall velocity $\dot{x}_0$ and the spin current u. Three important consequences of this interaction are found. First, spin-wave emission leads to a Landau-type damping of the current-induced domain wall motion toward restoring the solution $\dot{x}_0=u$, where spin angular momentum is perfectly transferred from the conduction electrons to the domain wall. Second, the interaction leads to a modification of the domain wall width and mass, proportional to the kinetic energy of the domain wall. Third, the coupling by the electrical current between the domain wall and the spin waves leads to temperature-dependent effective wall mass.

DOI: 10.1103/PhysRevB.79.174404 PACS number(s): 75.60.Ch, 75.30.Ds, 72.25.Pn, 85.75.−d

I. INTRODUCTION

The advent of giant magnetoresistance, magnetic tunnel junctions, and spin-transfer torque in magnetic heterostructures has led to proposals of novel applications in which magnetic domain walls are manipulated by electrical currents[1] instead of magnetic fields.[2,3] Experimental studies based on such concepts have been made possible by vast improvements in nanofabrication techniques, which allow for more precise control over domain wall nucleation and propagation.[4,5] While current-driven wall motion is an important means of realizing potential applications, the threshold current for such motion still remains prohibitively high for use in integrated circuits.[6] As a consequence, strategies are being actively sought to simultaneously achieve low threshold current densities in combination with high-speed domain wall motion.

From the point of view of fundamental physics, current-driven domain wall motion has attracted much interest because it associates a complex spin-dependent transport problem with nonlinear magnetization dynamics. This is equally true for ferromagnets based on 3*d* transition metals, such as iron, nickel, cobalt, and associated alloys, as for dilute magnetic semiconductors such as (Ga,Mn)As. From a theoretical perspective, the problem lies in computing the correct torques exerted on the magnetization by the conduction-electron spins. If one assumes that the conduction-electron spins, propagating with an effective drift velocity $\boldsymbol{u}$, track perfectly the local magnetization along their passage through the domain wall, one finds an additional torque on the magnetization $\boldsymbol{M}$ of the form

$$\boldsymbol{T}_{\rm a} = -(\boldsymbol{u}\cdot\nabla)\boldsymbol{M}, \qquad (1)$$

which is often referred to as the "adiabatic" contribution of spin transfer. This term is well understood and has been reproduced from different transport theories.[7–11] The magnitude of the effective drift velocity is given by $u = jPg\mu_B/(2eM_s)$, where j is the charge current density, P is the spin polarization, μ_B is the Bohr magneton, e is the electronic charge, and M_s is the saturation magnetization. An outstanding problem of importance concerns the origin of the so-called "nonadiabatic" contribution[12]

$$\boldsymbol{T}_{\rm na} = \frac{\beta}{M_s}\boldsymbol{M}\times[(\boldsymbol{u}\cdot\nabla)\boldsymbol{M}], \qquad (2)$$

which has been found to be necessary to describe some experimental data. M_s is the saturation magnetization and the dimensionless coefficient β characterizes the magnitude of the nonadiabatic contribution.

It is possible to gain good insight into the physics of current-driven motion through the one-dimensional (1D) model (1DM) for domain wall dynamics. This model was much studied in the 1970s (Refs. 13 and 14) and later adapted to the case of exchange torques due to coupling to conduction electrons by Berger[15–17] and Tatara and Kohno.[7] The 1DM is derived from the Landau-Lifshitz-Gilbert (LLG) equation of motion for magnetization dynamics with the current-driven terms,[18]

$$\frac{\partial \boldsymbol{M}}{\partial t} = \gamma_0\boldsymbol{H}_{\rm eff}\times\boldsymbol{M} + \frac{\alpha}{M_s}\boldsymbol{M}\times\frac{\partial\boldsymbol{M}}{\partial t} + \boldsymbol{T}_{\rm a} + \boldsymbol{T}_{\rm na}, \qquad (3)$$

where γ_0 is the gyromagnetic constant and α is the Gilbert damping constant. A critique concerning the relevance of Gilbert damping for domain wall motion has recently been presented by Stiles *et al.*[19] By assuming that the domain wall shape remains rigid during propagation, it is possible to parametrize the dynamics in terms of only two conjugated coordinates: the domain wall position x_0 and its conjugate momentum p. By assuming that the external forces acting on the wall are sufficiently weak such that the wall shape remains rigid, the equations of motion in this limit can be written as[7]

$$\frac{dp}{dt} = -\frac{2\alpha K_\perp m}{S}\frac{dx_0}{dt} + \frac{2\beta K_\perp m}{S}u + f_{\text{ze}} + f_{\text{pin}}, \quad (4)$$

$$\frac{dx_0}{dt} = u + \frac{\alpha S}{2K_\perp m}\frac{dp}{dt} + \frac{p}{m}, \quad (5)$$

where m is the domain wall mass, S is the spin angular momentum at each individual magnetic site, and $K_\perp$ is the transverse anisotropy energy. The external magnetic field and a pinning potential, due to intrinsic defects or artificial pinning centers, generate the additional forces $f_{\text{ze}}+f_{\text{pin}}$ on the domain wall, respectively. The first equation (4) relates the domain wall acceleration dp/dt to the total force. The second equation (5) relates the domain wall velocity to the domain wall momentum. Neglecting damping, one sees that p is related to the relative velocity $\dot{x}_0-u$ through the Döring mass of the domain wall $m=p/(\dot{x}_0-u)$.

The importance of the nonadiabatic "β term" is made explicit in Eq. (4), where its contribution as an effective magnetic field can be immediately seen. It has been shown in previous studies that the existence of β leads to different qualitative dynamics for the wall motion.[7] If $2\beta \geq \sqrt{H_p/H_\perp}$ with H_p being the extrinsic pinning field and $H_\perp$ being the transverse anisotropy field the domain wall depins for $u > \lambda\gamma_0 H_p/2\beta$. Therefore in the weak pinning limit, the larger is the β term, the smaller is the critical current density.[20]

However, as we have indicated above, the physical origin of this nonadiabatic term is still an open issue subject to spirited debate.[21] In one line of inquiry, different authors have sought to associate β with the viscous damping coefficient α since both parameters describe dissipative processes.[22] Barnes and Maekawa[23] contended that β and α are equal because of Galilean invariance, while Kohno *et al.*,[24] Duine *et al.*,[22] and Piechon and Thiaville[25] found that β and α are not equal in general. In a different picture, Tatara and Kohno[7] associated β with ballistic domain wall resistance, which is independent of α and depends only on the transport properties of the system. Much of the difficulty in reaching a consensus is therefore related to the complexity in defining the β term theoretically and in measuring it experimentally.

The present study is motivated by the hypothesis that the interaction between the domain wall and spin waves produces a term similar to the nonadiabatic term but in the presence of *only* the adiabatic component of spin transfer. The role of spin waves on field-driven domain wall dynamics has been examined by a number of authors in the past,[26–29] but their role on current-driven wall dynamics has not been studied in much detail theoretically. While most theories on the β term have focused on the transport properties of the conduction electrons, few studies have considered the motion of the nonequilibrium magnetization by taking into account the fluctuations. Nevertheless, the interplay between spin waves and the domain wall should be important for at least two reasons. First, thermal spin waves account for a decrease in the magnetization which can be important if the system temperature approaches the Curie temperature. This is certainly the case in dilute magnetic semiconductors. Second, the spin waves act as a thermal bath with which energy can be exchanged with the domain wall. Indeed, the importance of spin waves as a channel for energy dissipation in magnetic system has long been recognized. In the context of the ferromagnetic resonance, for example, two-, three- and four-magnon processes have been shown to be crucial for explaining resonance linewidths of ferromagnetic insulators.[30] In the context of domain wall motion, Bouzidi and Suhl[28] showed that power is diverted from the domain wall motion through the amplification of some thermal spin waves.

Recent experimental studies suggest that current-induced domain wall motion may depend strongly on the temperature.[31–36] Experiments on current-induced domain wall motion in metallic devices are generally performed at room temperature, but recently several measurements have been reported over a range of temperature from several dozens to a few hundreds of degrees kelvin.[32] Studies on the temperature dependence are likely to bring detailed information on the current-induced domain wall dynamics. The actual temperature of a ferromagnetic wire along which a charge current flows is generally modified by Joule heating and may vary much from one sample to another depending on the efficiency with which heat is drained out. As the current density required for driving a domain wall in a ferromagnetic metal is usually quite high, $j \approx 10^{12}$ A/m^2, the increase in the temperature due to Joule heating may even approach the Curie temperature T_c,[35] which would lead to large changes in the magnetization. Similar heating effects may also appear in nanowires involving magnetic semiconductors.[33,34] Laufenberg *et al.*[32] found current-driven domain wall motion to be less efficient by 50% for temperature increases of 200 K. These authors suggested that this loss of efficiency is due to the excitation of spin waves.

In this paper, we study the role of spin waves on current-driven domain wall motion by extending the approach used by Bouzidi and Suhl,[28] which associates some basic ideas from the theory of solitons[37] with spin-wave theory.[26,38,39] The coupling between the domain wall and the thermal bath of the spin waves, which originates from the kinetic part of the spin Lagrangian,[40,41] has a number of consequences on the current-induced domain wall motion. It leads to a different dissipation channel, whereby magnons can be absorbed or emitted as the domain wall propagates. This dissipation channel relaxes the domain wall dynamics toward the solution $\dot{x}_0=u$, where the domain wall velocity and the conduction-electron spin current are identical and Galilean invariance is restored. This dissipation process is somewhat analogous to Landau damping in plasmas.[42] The coupling between the spin waves and the current-driven domain wall also results in stochastic forces in addition to damping. These stochastic forces are weakly correlated at the time scale of the domain wall motion and therefore can be treated as white noise. In the absence of Galilean invariance, $u \neq \dot{x}_0$, the flow of the spin current across the wire leads to a reduction in the domain wall width, which renormalizes the system energy. This renormalization can be reinterpreted as a modification of the domain wall mass, which becomes temperature dependent through the interaction with the spin waves.

This paper is organized as follows. The spin-wave eigenmodes of the domain wall are determined in Sec. II. In Sec. III the one-dimensional (1D) model of current-driven domain

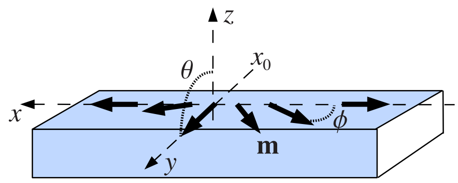

FIG. 1. (Color online) Geometry. The wire is along the x axis and the static domain wall profile is in the (x,y) plane because of a strong perpendicular anisotropy along z. The spherical polar coordinates are defined with respect to the z direction.

wall dynamics is generalized to account for spin waves. In Sec. IV, the damping of domain wall motion through radiation of magnons is presented. This radiation leads to both α-like and β-like terms, which are both proportional to the domain wall kinetic energy $p^2/2m$. The change in domain wall width by the electrical current is calculated in Sec. V and subsequently interpreted as a renormalization of the domain wall mass. The response of the spin waves to the domain wall displacement and to the spin-transfer torque is investigated in Sec. VI. The renormalization of the domain wall mass, as a result of this response, is then estimated numerically. In Sec. VII, we present some discussion and concluding remarks, as well as offer suggestions for new experiments that are designed to test the main results of our theory. The Green's functions used for our calculations and the integral equation used for determining the spin-wave response are presented in the Appendix.

II. EIGENMODES OF A BLOCH DOMAIN WALL

We consider a ferromagnetic wire lying along the x axis with an axis of easy anisotropy K_u along the x direction and an axis of hard perpendicular (transverse) anisotropy $K_\perp$ along the z direction (see Fig. 1). The orientation of the localized spins is described in spherical coordinates within a continuum approximation by means of a field $\boldsymbol{m}(x,t) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, where $\theta(x,t)$ and $\phi(x,t)$ have a space and time dependence. In the absence of a conduction-electron charge current, the magnetic energy $\mathcal{H}$ of the system is

$$\mathcal{H} = \int \frac{d^3r}{a^3}\{A[(\nabla\theta)^2 + \sin^2\theta(\nabla\phi)^2] - K_u \sin^2\theta\cos^2\phi + K_\perp\cos^2\theta\}, \tag{6}$$

where A denotes the exchange coupling. In the following we assume $K_\perp \gg K_u$. This condition applies well to thin Permalloy nanowires whose in-plane (magnetocrystalline) anisotropies are very small compared to the perpendicular (demagnetizing) energy. The anisotropy constant K_u describes the shape anisotropy in the plane of the wire and is as small as a few oersteds, whereas the anisotropy constant $K_\perp$ describes the demagnetizing field and is about $4\pi M_s \approx 10$ kOe.

The equilibrium magnetic configuration can be found by minimizing the energy functional by using the conditions

$$\left.\frac{\delta\mathcal{H}_m}{\delta\theta}\right|_{\theta_0,\phi_0} = 0, \quad \left.\frac{\delta\mathcal{H}_m}{\delta\phi}\right|_{\theta_0,\phi_0} = 0, \tag{7}$$

which lead to

$$\theta_0 = \frac{\pi}{2}, \tag{8}$$

$$A\frac{\partial^2\phi_0}{\partial x^2} = K_u \sin\phi_0\cos\phi_0. \tag{9}$$

The solution corresponds to the well-known Bloch domain wall with a characteristic width of $\lambda = \sqrt{A/K_u}$ and energy $\sigma = 4K_u N_{\rm dw}$, where $N_{\rm dw}$ denotes the number of magnetic sites inside the domain wall. The domain wall profile can be expressed in terms of the spatial coordinate as $\sin\phi_0 = \mathrm{sech}[(x-x_0)/\lambda]$ and $\cos\phi_0 = -\tanh[(x-x_0)/\lambda]$.

To account for thermal fluctuations, we consider small deviations $(\delta\theta, \delta\phi)$ about the static configuration (θ_0, ϕ_0). We expand $\mathcal{H}$ up to the second order with respect to $\delta\theta$ and $\delta\phi$ to obtain

$$\delta\mathcal{H} = \frac{K_u}{a^3}\int d^3r\{\delta\theta(\mathcal{D}+\kappa)\delta\theta + \delta\phi\mathcal{D}\delta\phi\}. \tag{10}$$

In agreement with earlier works,[26,28,29] we find that the energy of the thermal fluctuations is described by a Schrödinger-type operator $\mathcal{D} = -\lambda^2\partial_x^2 - 2\,\mathrm{sech}[(x-x_0)/\lambda] + 1$ with $\kappa = K_\perp/K_u$. The eigenvalues of $\mathcal{D}$ are 0 and $\omega_{\boldsymbol{k}} = 1 + \boldsymbol{k}^2\lambda^2$. The zero-eigenvalue solution $\xi_{\rm loc}$,

$$\xi_{\rm loc}(\boldsymbol{r}) = \frac{1}{\sqrt{2N_{\rm dw}}}e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\,\mathrm{sech}\left(\frac{x-x_0}{\lambda}\right), \tag{11}$$

corresponds to the Goldstone mode of the system since the energy of the static wall is independent of its position x_0. In other words the $\xi_{\rm loc}$ part of $\delta\phi$ contains no energy $\xi_{\rm loc}\mathcal{D}\xi_{\rm loc} = 0$. We can avoid expanding $\delta\phi$ on the Goldstone mode by elevating domain wall position to a dynamical collective coordinate $x_0(t)$.[37] The system is not rotationally invariant about the wire axis because of the strong perpendicular anisotropy $K_\perp$. As such, the $\xi_{\rm loc}$ part of $\delta\theta$ carries a finite energy and the wave function $\xi_{\rm loc}$ corresponds to a bound state of the system. In the following the amplitude of this bound state will be noted as $c_{\rm loc}(t)$. The nonzero eigenvalues of operator $\mathcal{D}$ correspond to the propagating waves $\xi_{\boldsymbol{k}}(\boldsymbol{r})$,

$$\xi_{\boldsymbol{k}}(\boldsymbol{r}) = \frac{1}{\sqrt{\omega_{\boldsymbol{k}}N}}e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\left[\tanh\left(\frac{x-x_0}{\lambda}\right) - ik_x\lambda\right]. \tag{12}$$

We have noted N as the total number of magnetic sites in the sample. The wave functions $\xi_{\boldsymbol{k}}$ form an orthonormal set

$$\int\frac{d^3r}{a^3}\xi_{\boldsymbol{k}}^*\xi_{\boldsymbol{m}} = \delta_{\boldsymbol{k},\boldsymbol{m}}, \tag{13}$$

which, in turn, are orthogonal to the bound-state wave function $\xi_{\rm loc}$, i.e.,

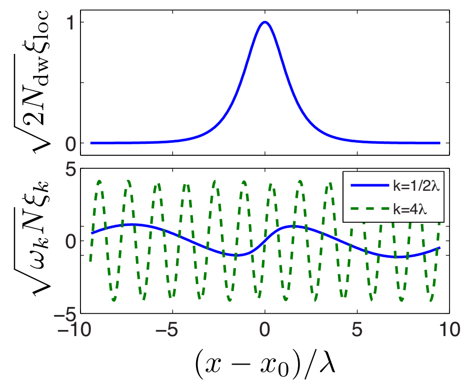

FIG. 2. (Color online) Wave functions $\sqrt{2N_{\rm dw}}\xi_{\rm loc}(\boldsymbol{r})$ and $\sqrt{\omega_k N}\xi_k(\boldsymbol{r})$ about the domain wall at x_0 for $k_x = 1/2\lambda$ and $k_x = 4/\lambda$.

$$\int \frac{d^3r}{a^3}\xi_k^*\xi_{\rm loc} = 0. \quad (14)$$

The wave functions $\xi_{\rm loc}$ and ξ_k are represented in Fig. 2.

It is convenient to expand the small angle deviations $\delta\theta$ and $\delta\phi$ in terms of the eigenfunctions ξ_k via the complex-valued variables d_k through the transformation $\delta\phi + i\delta\theta = ic_{\rm loc}\xi_{\rm loc} + \Sigma_k d_k\xi_k$. Using this notation, we note that the Hamiltonian can be written as $\mathcal{H} = \sigma + (\sigma/4)\Sigma_k\{(\omega_k + \kappa/2)d_k^* d_k - (\kappa/4)(d_k d_{-k} + d_k^* d_{-k}^*)\}$. This is not a quadratic Hamiltonian because of the finite perpendicular anisotropy $K_\perp$, which leads to elliptical spin precession. To diagonalize this Hamiltonian, we follow the usual prescription by means of the Bogoliubov transformation $c_k = u_k^+ d_k + u_k^- d_{-k}^*$ with

$$u_k^\pm = \sqrt{\frac{\omega_k + \kappa/2 \pm \hbar\Omega_k/K_u}{2\hbar\Omega_k/K_u}}, \quad (15)$$

where the frequency Ω_k is defined as

$$\left(\frac{\hbar\Omega_k}{K_u}\right)^2 = \omega_k(\omega_k + \kappa). \quad (16)$$

By replacing d_k and d_k^* with the magnon operators c_k and c_k^*, we obtain the quadratic spin-wave Hamiltonian

$$\delta\mathcal{H} = K_\perp c_{\rm loc}^2 + \sum_k \hbar\Omega_k c_k^* c_k, \quad (17)$$

where the spin-wave energy is $\hbar\Omega_k$. The mode c_k has two components $c_k = c_k^{\rm def} + c_k^{\rm th}$, which describe the wall deformation and the thermal propagating spin-wave excitations.

Next, we quantize the system by turning the complex variables $c_k^{\rm th}$ and $c_k^{\rm th*}$ into the boson operators $\hat{c}_k$ and $\hat{c}_k^\dagger$, which obey the usual bosonic commutation relations. For the sake of clarity, we will find it convenient to use the variables $\phi_k = c_k + c_{-k}^*$ and $\theta_k = (1/i)(c_k - c_{-k}^*)$ and their corresponding operators $\hat{\phi}_k = \hat{c}_k + \hat{c}_{-k}^\dagger$ and $\hat{\theta}_k = (1/i)(\hat{c}_k - \hat{c}_{-k}^\dagger)$. The small angle deviations $\delta\phi$ and $\delta\theta$ are then expressed in terms of ϕ_k and θ_k as

$$\delta\phi(\boldsymbol{r}) = \sum_k \phi_k \nu_k^\phi \xi_k(\boldsymbol{r}), \quad (18)$$

$$\delta\theta(\boldsymbol{r}) = c_{\rm loc}\xi_{\rm loc}(\boldsymbol{r}) + \sum_k \theta_k \nu_k^\theta \xi_k(\boldsymbol{r}). \quad (19)$$

The parameters $\nu_k^\phi = (u_k^+ + u_k^-)/2$ and $\nu_k^\theta = (u_k^+ - u_k^-)/2$ represent the ellipticity of the spin precession. If the system were rotationally invariant about the wire direction $K_\perp = 0$, then spin precession would be circular $\nu_k^\phi = \nu_k^\theta = 1/2$.

Lastly, it is convenient to renormalize the bound-state amplitude $c_{\rm loc}$ and introduce a new variable p as

$$p = -\frac{S\sqrt{2N_{\rm dw}}c_{\rm loc}}{\lambda}. \quad (20)$$

We will see in Sec. III that $p(t)$ represents the domain wall kinetic momentum.

III. GENERALIZED 1D MODEL OF BLOCH WALL DYNAMICS

A. 1D model without spin waves

As we will show in subsequent sections, the deformation of the domain wall due to spin-transfer torques is described by both the bound state $c_{\rm loc}$ and the propagating states c_k. However, we will disregard the spin-wave modes c_k to begin with in this section and discuss rather the role of the sole bound-state amplitude $c_{\rm loc} \equiv p(t)$ on the domain wall dynamics. We show that we recover the usual one-dimensional model of Bloch wall dynamics without spin waves.

In the absence of the nonadiabatic spin-transfer term, it is possible to derive the equations of motion using a Lagrangian formalism. This is particularly well adapted to the present problem in which the magnetic system is subject to constraints related to the presence of the domain wall. The total Lagrangian for the magnetic system is the difference between a "kinetic" (or Berry phase) term,

$$\mathcal{L}_{\rm kin}(u=0) = S\int \frac{d^3r}{a^3}(1 - \cos\,\theta)\partial_t\phi, \quad (21)$$

and the magnetic energy of the system $\mathcal{H}$. The inclusion of the adiabatic spin-transfer term appears as a moving reference frame at a velocity equal to the effective drift velocity of the spin current u. This is accounted for by replacing the time derivative in the kinetic part of the Lagrangian with a convective derivative,[40,43]

$$\mathcal{L}_{\rm kin} = S\int \frac{d^3r}{a^3}(1 - \cos\,\theta)(u\partial_x + \partial_t)\phi. \quad (22)$$

To zeroth order in the deformation, the kinetic part of the Lagrangian does not contribute to dynamics and can be neglected. The first-order term of $\mathcal{L}_{\rm kin}$ with respect to the deformation is

$$\mathcal{L}_{\rm kin}^{(1)} = S\int \frac{d^3r}{a^3}c_{\rm loc}\xi_{\rm loc}(u\partial_x + \partial_t)\phi_0,$$

$$= -p(u - \dot{x}_0). \quad (23)$$

The overall magnetic energy is the sum of the static domain wall energy σ and the dipolar energy $K_\perp c_{\rm loc}^2$. The latter can be reinterpreted as the kinetic energy $p^2/2m$ of the domain wall,

$$\mathcal{H}_m = \sigma + K_\perp c_{\rm loc}^2 = \sigma + \frac{p^2}{2m},$$

where $m = N_{\rm dw} S^2 / K_\perp \lambda^2$ is the Döring mass. Finally the full Lagrangian is obtained as

$$\mathcal{L} = -p(u - \dot{x}_0) - \sigma - p^2/2m. \tag{24}$$

Gilbert or viscous damping can be accounted for by including the dissipation function

$$\mathcal{F} = \alpha S \int \frac{d^3 r}{a^3} (\dot{\theta}^2 + \dot{\phi}^2 \sin^2 \theta) \tag{25}$$

in the equations of motion. By assuming the "rigid" domain wall profile $\theta = \theta_0(t)$ and $\phi = \phi_0(t)$, the dissipation function $\mathcal{F}$ can be readily rewritten as

$$\mathcal{F} = \alpha S \left(\frac{\dot{p}^2 \lambda^2}{2 S^2 N_{\rm dw}} + \frac{2 N_{\rm dw} \dot{x}_0^2}{\lambda^2} \right). \tag{26}$$

The equations of motion for the domain wall coordinates $q = (x_0, p)$ are obtained by means of the Euler-Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial q} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{1}{2} \frac{\partial \mathcal{F}}{\partial \dot{q}}. \tag{27}$$

By combining the Lagrangian functional $\mathcal{L}$ (24) and the dissipation function $\mathcal{F}$ (26), the Euler-Lagrange equations lead to

$$\frac{dp}{dt} + \frac{\alpha 2 K_\perp}{S} m \frac{dx_0}{dt} = 0, \tag{28}$$

$$\frac{dx_0}{dt} - u - \frac{\alpha S}{m 2 K_\perp} \frac{dp}{dt} = \frac{p}{m}. \tag{29}$$

Thus, the well-known equations of motion of the one-dimensional Bloch domain wall, Eqs. (4) and (5), are recovered. We conclude that the bound-state amplitude $p(t)$ can be interpreted as the domain wall momentum as long as the propagating spin waves are neglected.

B. Interaction between domain wall and propagating modes

The expansion of the full Lagrangian $\mathcal{L}$ up to the first order in the spin waves (23) involves only the bound state and does not depend on the propagating waves. The interaction between the propagating waves and the domain wall arises from the second-order expansion

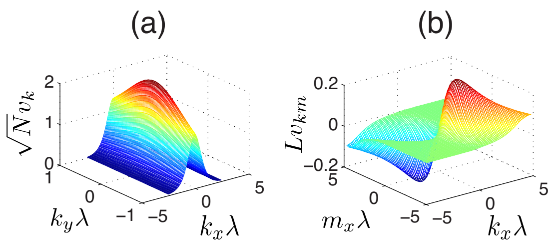

FIG. 3. (Color online) (a) $\sqrt{N} v_{\boldsymbol{k}}$ as a function of k_x and k_y. (b) $L v_{\boldsymbol{km}}$ as a function of k_x and m_x. $K_\perp / K_u = 57$ for both graphs.

$$\mathcal{L}_{\rm kin}^{(2)} = \int \frac{d^3 r}{a^3} S \delta\theta (u \partial_x + \partial_t) \delta\phi,$$

$$\mathcal{L}_{\rm kin}^{(2)} = S \sum_{\boldsymbol{k}} \frac{1}{4} \theta_{\boldsymbol{k}} \dot{\phi}_{-\boldsymbol{k}} + S(u - \dot{x}_0) \times \left[-\sum_{\boldsymbol{k}} v_{\boldsymbol{k}} \frac{p}{S} \phi_{\boldsymbol{k}} + \sum_{\boldsymbol{k} \neq \boldsymbol{m}} v_{\boldsymbol{km}} \theta_{\boldsymbol{k}} \phi_{\boldsymbol{m}} + \sum_{\boldsymbol{k}} \frac{k_x}{2} c_{\boldsymbol{k}}^* c_{\boldsymbol{k}} \right]. \tag{30}$$

The coefficient $v_{\boldsymbol{k}}$ describes a coupling between the bound state ξ_0 and the propagating waves,

$$v_{\boldsymbol{k}} = \frac{\pi}{2} \frac{1}{\sqrt{\omega_{\boldsymbol{k}} N}} \nu_{\boldsymbol{k}}^\phi \,{\rm sech}\left(\frac{k_x \lambda \pi}{2} \right). \tag{31}$$

The coefficient $v_{\boldsymbol{km}}$ describes a coupling between the different propagating waves,

$$v_{\boldsymbol{km}} = \nu_{\boldsymbol{k}}^\theta \nu_{\boldsymbol{m}}^\phi \frac{i\pi}{2} \frac{(k_x\lambda)^2 - (m_x\lambda)^2}{L\sqrt{\omega_{\boldsymbol{k}} \omega_{\boldsymbol{m}}}} {\rm csch}\left(\frac{\pi\lambda(k_x + m_x)}{2} \right) \delta_{\boldsymbol{k}_\parallel, -\boldsymbol{m}_\parallel} \tag{32}$$

with $L = N_x a$ as the length of the wire. Coefficients $v_{\boldsymbol{k}}$ and $v_{\boldsymbol{km}}$ are shown in Fig. 3.

$v_{\boldsymbol{k}}$ becomes negligible for $k_x > 1/\lambda$, which means that the term $-(u - \dot{x}_0) \Sigma_{\boldsymbol{k}} v_{\boldsymbol{k}} p \phi_{\boldsymbol{k}}$ will mainly couple the domain wall to spin waves with a wave vector k_x on the order of $1/\lambda$. In addition, the coefficient $v_{\boldsymbol{km}}$ is large for long-wavelength spin waves $k < 1/\lambda, m < 1/\lambda$ and is roughly proportional to $k_x - m_x$. Thus the coupling term $S(u - \dot{x}_0) \Sigma_{\boldsymbol{km}} v_{\boldsymbol{km}} \theta_{\boldsymbol{k}} \phi_{\boldsymbol{m}}$ will lead to a significant interaction with spin waves having $k_x \sim -m_x \sim \pm 1/\lambda$. In some sense, the latter coupling term represents reflection of the propagating spin waves from the domain wall.

The quadratic term $S(u - \dot{x}_0) \Sigma_{\boldsymbol{k}} (k_x/2) \hat{c}_{\boldsymbol{k}}^\dagger \hat{c}_{\boldsymbol{k}}$ represents a shift in the dispersion relation of the magnons. As such, the frequencies of the magnons depend on the relative velocity between the spin current and the domain wall,

$$\epsilon_{\boldsymbol{k}} \to \epsilon_{\boldsymbol{k}} - \frac{S}{2} k_x (u - \dot{x}_0). \tag{33}$$

This shift may be interpreted as a Doppler effect, which was originally put forward by Lederer and Mills[44] and found recently in experiment by Vlaminck and Bailleul.[45] Some consequences of this Doppler shift have already been investigated so far, e.g., the excitation of monodomain structures by

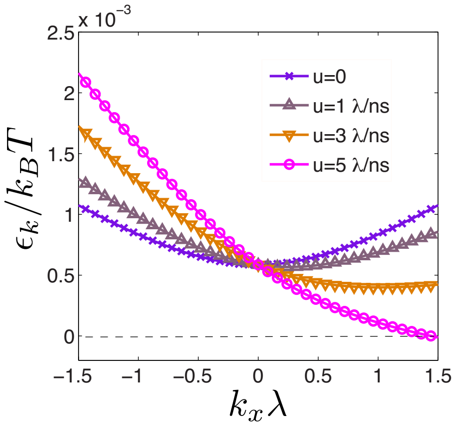

FIG. 4. (Color online) Dispersion relation of the magnons as a function of the spin-current velocity u for $K_\perp/K_u=57$ and $\hbar/K_u=1.88$ ns.

a sole dc electrical current[40] or as the enhancement of dissipation by spin-transfer torque.[46] An example of the current-induced Doppler effect on the spin-wave dispersion relations is shown in Fig. 4. We note that the current-driven terms can lead to a soft mode with a nonzero wave vector k_c, as indicated by the dashed line in Fig. 4 for $u=5\lambda/\text{ns}$ for $k_c\lambda \simeq 1.5$. As pointed out by Shibata *et al.*,[40] this would lead to an instability in a uniformly magnetized ground state whereby the nucleation of domains of size $\sim\pi/k_c$ is favored.

Combining the kinetic part of the Lagrangian and the magnetic energy, the full Lagrangian $\mathcal{L}$ describing the magnetic system is found to be

$$\mathcal{L} = S\sum_k \frac{1}{4}\theta_k\dot{\phi}_{-k} + S(u-\dot{x}_0)\left[-\frac{p}{S} - \sum_k v_k \frac{p}{S}\phi_k + \sum_k \frac{k_x}{2}c_k^* c_k + \sum_{k\neq -m} v_{km}\theta_k\phi_m\right] - \sigma - \frac{p^2}{2m} - \sum_k \hbar\Omega_k c_k^* c_k. \quad (34)$$

The classical coordinates of the domain wall are coupled to the propagating spin waves through the interaction potential

$$\mathcal{V} = S(u-\dot{x}_0)\left[\sum_k v_k \frac{p}{S}\phi_k - \sum_{km} v_{km}\theta_k\phi_m\right]. \quad (35)$$

This potential must exist because the wave functions ξ_0 and ξ_k do not diagonalize the kinetic part of the Lagrangian $\mathcal{L}_{\text{kin}}$. As such, a finite difference $u\neq\dot{x}_0$ between the spin-current velocity u and the domain wall velocity $\dot{x}_0$ will always give rise to a coupling between the propagating spin waves and the domain wall. However, if the spin transfer from the adiabatic torque is achieved with an effective drift velocity such that $u=\dot{x}_0$, then the interaction potential $\mathcal{V}$ will vanish identically and, in that case, the domain wall and the spin waves will be completely decoupled. We point out that the solution $\dot{x}_0=u$ is actually satisfied if the β coefficient in the one-dimensional model is identical to the Gilbert coefficient α or, in other words, the conservative and nonconservative dynamics of the system are invariant under a Galilean transformation.

C. Force and torque

In the following, we seek to generalize the 1DM by the inclusion of the interacting potential (35) which couples the spin-wave modes and the domain wall. The Euler-Lagrange equations (28) and (29) become modified by the latter and therefore involve new terms with respect to the magnons.

By definition, the force F exerted on the domain wall is equal to the time derivative of the domain wall momentum dp/dt. The Euler-Lagrange equation

$$\frac{dp}{dt} = -\frac{\partial\mathcal{H}}{\partial x_0} - \frac{d}{dt}\frac{\partial\mathcal{V}}{\partial\dot{x}_0} \quad (36)$$

indicates that F originates in both the magnetic energy $\mathcal{H}$ and the interaction potential $\mathcal{V}$. Let us first consider the contribution F_H from the magnetic energy $\mathcal{H}$,

$$F_H = -\frac{\partial\mathcal{H}}{\partial x_0}. \quad (37)$$

This force may be rewritten as

$$F_H = -4K_u\int\frac{d^3r}{a^3}\delta m_z(x)\partial_x \operatorname{sech}^2(x-x_0), \quad (38)$$

where δm_z represents the decrease in the longitudinal component of the magnetization $\delta m_z=(\delta\theta^2+\delta\phi^2)/2$ and is nonzero at finite temperatures due to thermal excitations. By inspection of Eq. (38), we observe that F_H is only finite if δm_z is odd with respect to the domain wall position $\delta m_z(x_0-x_1)=-\delta m_z(x_0+x_1)$. This can be the case if an electrical current flows through the domain wall and breaks the symmetry of the system.

Let us now consider the force $-d/dt(d\mathcal{V}/d\dot{x}_0)$ that arises from the interaction potential $\mathcal{V}$. This force may be divided into three different components: F_j, F_{def}, and F_{stoc}. The force F_j,

$$F_j = S\frac{d}{dt}\sum_{km} v_{km}\langle\hat{\theta}_k\hat{\phi}_m\rangle, \quad (39)$$

originates from the coupling between the different spin-wave modes induced by the electrical current. Because $\langle\hat{\theta}_k\hat{\phi}_m\rangle$ depends on the statistics of the magnons, the force F_j is expected to depend on the temperature. The force F_{def} is related to the deformation modes c_k^{def} (see Sec. II). By writing $\phi_k^{\text{def}}=c_k^{\text{def}}+c_{-k}^{\text{def}*}$, we find

$$F_{\text{def}} = -\frac{d}{dt}\sum_k v_k p\,\phi_k^{\text{def}}. \quad (40)$$

In contrast to F_j and F_{def}, the force F_{stoc} is not deterministic but stochastic. Its average value vanishes $\langle F_{\text{stoc}}\rangle=0$ but its autocorrelation function is finite $\langle F_{\text{stoc}}(t)F_{\text{stoc}}(t')\rangle\neq 0$. The force $F_{\text{stoc}}(t)$ may be rewritten as

$$F_{\text{stoc}} = -\frac{d}{dt}\sum_k v_k p\,\hat{\phi}_k. \quad (41)$$

By summing all these forces together, the total force F exerted on the domain wall is found to be

$$\frac{dp}{dt} = F_H + F_j + F_{\text{def}} + F_{\text{stoc}}. \tag{42}$$

It will be shown in the following that all the forces F_H, F_j, and F_{def} can be reinterpreted as a modification of the domain wall mass.

So far we have looked at the contribution of the spin waves to the force F but still not to the torque T. The torque T is derived by taking the Euler-Lagrange equations with respect to the momentum p,

$$\dot{x}_0 = \frac{p}{m} + u - \sum_{\boldsymbol{k}} v_{\boldsymbol{k}} \phi_{\boldsymbol{k}} (\dot{x}_0 - u). \tag{43}$$

In addition to the spin-transfer drift velocity u and to p/m, which represents the torque due to the demagnetizing field, Eq. (43) involves a stochastic torque T_{stoc},

$$T_{\text{stoc}} = (\dot{x}_0 - u) \sum_{\boldsymbol{k}} v_{\boldsymbol{k}} \hat{\phi}_{\boldsymbol{k}}. \tag{44}$$

This torque is induced by the spin waves, which increases as a function of the relative velocity $\dot{x}_0 - u$.

IV. SPIN-WAVE EMISSION

In this section, we present a detailed analysis of the consequences of spin-wave emission as a result of domain wall motion. In particular, we show that such processes lead to a dissipation mechanism that drives the domain wall velocity toward the spin-current velocity, thereby restoring Galilean invariance in the system. We show that spin-wave emission also leads to a stochastic force on the domain wall.

A. Dissipation function

According to Eqs. (41) and (44), spin waves exert a nondeterministic force and torque on the domain walls. This is because the domain walls are sensitive to the fluctuations of the spin waves through the interaction potential

$$\mathcal{V}_{\text{int}} = (u - \dot{x}_0) p \sum_{\boldsymbol{k}} v_{\boldsymbol{k}} (\hat{c}_{\boldsymbol{k}} + \hat{c}^{\dagger}_{-\boldsymbol{k}}). \tag{45}$$

We know by the fluctuation-dissipation theorem that these fluctuations necessarily lead to dissipation. As we have not taken this dissipation into account yet, our 1DM with the forces F_H, F_j, F_{def}, F_{stoc}, and the torque T_{stoc} is not fully consistent.

The spin current supplies energy to the domain wall motion by increasing the kinetic energy $p^2/2m$. However a part of this energy is lost by the domain wall and is transferred to the lattice through a series of relaxation processes. The transfer of energy to the lattice accompanying a domain wall motion is due to various channels of relaxation.[47] One channel of relaxation starts with the emission of magnons by the excited domain wall.

Caldeira and Leggett,[48] following on from the seminal work and Feynman and Vernon,[49] showed irreversibility to arise when a moving particle is coupled to numerous degrees of freedom. In our case, the magnetic domain wall represents the moving particle, which propagates through a dissipative environment represented by the thermal spin waves. The domain wall is described by its position x_0 and its kinetic momentum p, whereas the spin-wave environment is described by a set of oscillators,

$$\mathcal{H}_{\text{bath}} = \sum_{\boldsymbol{k}} \hbar \Omega_{\boldsymbol{k}} \hat{c}^{\dagger}_{\boldsymbol{k}} \hat{c}_{\boldsymbol{k}}. \tag{46}$$

The bath of magnons is assumed to be always at thermal equilibrium, which is maintained through very fast three- and four-magnon processes and due to the interaction with phonons. The domain wall and the spin-wave environment are coupled to each other through the "linear" interaction potential $\mathcal{V}_{\text{int}}$. Notice the similarity between the interaction $\mathcal{V}_{\text{int}}$ and the interaction between electrons and phonons in the metals. The present system is actually formally equivalent to the dissipative system in Caldeira and Leggett's general theory, which itself is in agreement with the fluctuation-dissipation theorem.[50] Notice that a coupling very close to $\mathcal{V}_{\text{int}}$ has already been considered by Thompson[41] who investigated the damped motion of vortices driven by a magnetic field.

The emission of a magnon $\boldsymbol{k}$ is represented by the term

$$\mathcal{V}^{\boldsymbol{k}}_{\text{em}} = (u - \dot{x}_0) p v_{\boldsymbol{k}} \hat{c}^{\dagger}_{\boldsymbol{k}}. \tag{47}$$

Similarly the absorption by the domain wall of a magnon $\boldsymbol{k}$ is represented by

$$\mathcal{V}^{\boldsymbol{k}}_{\text{ab}} = (u - \dot{x}_0) p v_{\boldsymbol{k}} \hat{c}_{\boldsymbol{k}}. \tag{48}$$

The energy transfer from the domain wall to the spin waves ensemble will correspond to the difference between the emission of magnons and the absorption of magnons.

In the following, the various states of the spin-wave ensemble will be denoted by $|E_n\rangle$ and the density of state of the spin waves will be represented by $\rho(E)$. By Fermi's golden rule, if the spin-wave ensembles were in the state $|E_n\rangle$, then the exchange of a magnon $\boldsymbol{k}$ between the domain wall and the bath of the spin waves would statistically decrease the domain wall energy by an amount $F_n(\boldsymbol{k})$ as

$$F_n(\boldsymbol{k}) = 2\pi\Omega_{\boldsymbol{k}} \Bigg[\sum_m |\langle E_m | \mathcal{V}^{\boldsymbol{k}}_{\text{em}} | E_n \rangle|^2 \delta(E_m - E_n - \hbar\Omega_{\boldsymbol{k}}) - \sum_m |\langle E_m | \mathcal{V}^{\boldsymbol{k}}_{\text{ab}} | E_n \rangle|^2 \delta(E_m - E_n + \hbar\Omega_{\boldsymbol{k}}) \Bigg],$$

which can be written explicitly as

$$F_n(\boldsymbol{k}) = 2\pi\Omega_{\boldsymbol{k}} \Bigg[\sum_m |(u - \dot{x}_0) p v_{\boldsymbol{k}}|^2 \delta(E_m - E_n - \epsilon_{\boldsymbol{k}}) - \sum_m |(u - \dot{x}_0) p v_{\boldsymbol{k}}|^2 \delta(E_m - E_n + \epsilon_{\boldsymbol{k}}) \Bigg]. \tag{49}$$

In reality the probability for the spin-wave ensemble to remain in the state $\langle E_n \rangle$ is equal to the Boltzmann factor $P_n = \exp(-\beta E_n)$. The magnons $\boldsymbol{k}$ thus contribute to a transfer of energy from the domain wall to the spin-wave ensembles by an amount $F(\boldsymbol{k}) = \Sigma_n P_n F_n(\boldsymbol{k})$. In the continuum limit $\Sigma_n E_n \to \int dE$, we find

$$F(\boldsymbol{k}) = \int_0^\infty dE\ \rho(E)f(E)F_E(\boldsymbol{k}), \qquad (50)$$

where $f(E)=\exp(-\beta E)$ and $F_E(\boldsymbol{k})$ is

$$F_E(\boldsymbol{k}) = 2\pi\Omega_{\boldsymbol{k}}|(u-\dot{x}_0)pv_{\boldsymbol{k}}|^2 \times \sum_m [\delta(E_m - E - \hbar\Omega_{\boldsymbol{k}}) - \delta(E_m - E + \hbar\Omega_{\boldsymbol{k}})].$$

In other words the energy lost by the domain wall through the transfer of a magnon $\boldsymbol{k}$ is

$$F(\boldsymbol{k}) = |(u-\dot{x}_0)p|^2\Omega_{\boldsymbol{k}}^2 v_{\boldsymbol{k}}^2 R(\boldsymbol{k}), \qquad (51)$$

where the function $R(\boldsymbol{k})$ is closely related to the spectral function of the environment and is given by

$$R(\boldsymbol{k}) = \int_0^\infty dE\ \rho(E)f(E)\frac{2\pi}{\Omega_{\boldsymbol{k}}}[\rho(E+\epsilon_{\boldsymbol{k}}) - \rho(E-\epsilon_{\boldsymbol{k}})]$$

with $\rho(E)$ being the density of states $\rho(E)=\Sigma_m\delta(E_m-E)$ of the spin-wave ensemble. The total energy lost by the domain wall per unit of time $\mathcal{F}=\Sigma_{\boldsymbol{k}}F_{\boldsymbol{k}}$ is finally obtained as

$$\mathcal{F} = \sum_{\boldsymbol{k}} |(u-\dot{x}_0)p|^2\Omega_{\boldsymbol{k}}^2 v_{\boldsymbol{k}}^2 R(\boldsymbol{k}). \qquad (52)$$

Let us introduce the dimensionless friction parameter $\eta = \Sigma_{\boldsymbol{k}}\Omega_{\boldsymbol{k}}^2 v_{\boldsymbol{k}}^2 R(\boldsymbol{k})$. The dissipation function can then be rewritten as

$$\mathcal{F} = \frac{2\eta}{N_{\mathrm{dw}}S}|(u-\dot{x}_0)p|^2. \qquad (53)$$

Within this approximation, retardation effects of spin waves are not included and therefore energy dissipation function is found to be local in time. This limit corresponds to the limit where the time scale of interest is much greater than the correlation time of the magnons.[41]

B. β term

To understand how this dissipation function affects current-induced domain wall motion, it is convenient to perform a Galilean transformation and use the local frame moving at the velocity of the spin current. The domain wall position in this frame is

$$q = x_0 - ut. \qquad (54)$$

The dissipation function $\mathcal{F}=-d\mathcal{H}/dt$ can be rewritten in terms of q as

$$\mathcal{F} = \frac{2\eta}{N_{\mathrm{dw}}S}|\dot{q}p|^2 \qquad (55)$$

and by treating q and p as independent variables, we find

$$\mathcal{F} = \frac{1}{2}\frac{\partial\mathcal{F}}{\partial\dot{q}}\dot{q}. \qquad (56)$$

If the system is translationally invariant (no extrinsic pinning), the Lagrangian describing the domain wall motion will not depend on the domain wall position q,

$$\mathcal{L} = -\mathcal{H} + \frac{\partial\mathcal{L}}{\partial\dot{q}}\dot{q}, \qquad (57)$$

in which case the time derivative of the domain wall Hamiltonian becomes

$$\frac{d\mathcal{H}}{dt} = \left(\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}} - \frac{\partial\mathcal{L}}{\partial q}\right)\dot{q}. \qquad (58)$$

By combining Eq. (58) and the definition of the dissipation function $\mathcal{F}$, we obtain

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}} - \frac{\partial\mathcal{L}}{\partial q} + \frac{1}{2}\frac{\partial\mathcal{F}}{\partial\dot{q}} = 0, \qquad (59)$$

which, after carrying out Eqs. (24) and (55), yields

$$\dot{p} + \frac{2\eta p^2}{N_{\mathrm{dw}}S}\dot{x}_0 = \frac{2\eta p^2}{N_{\mathrm{dw}}S}u. \qquad (60)$$

The dissipation by spin-wave emission gives rise to a damping term $2\eta p^2\dot{x}_0/N_{\mathrm{dw}}$ proportional to the domain wall velocity $\dot{x}_0$ and to a force $2\eta p^2 u/N_{\mathrm{dw}}S$ proportional to the spin current u. These are equivalent to a damping coefficient α_{sw} and to a coefficient β_{sw} expressed by

$$\alpha_{\mathrm{sw}} = \beta_{\mathrm{sw}} = \frac{\eta p^2}{N_{\mathrm{dw}}K_\perp m}. \qquad (61)$$

It is seen that the spin-wave contributions α_{sw} and β_{sw} are identical to each other and are also proportional to the domain wall kinetic energy $p^2/2m$.

The dissipation function generally tends to decrease during the motion and vanishes at steady state. In the case of Gilbert damping, which is described by the dissipation function $F/2SN_{\mathrm{dw}}=\alpha(\dot{x}_0/\lambda)^2(1+\alpha^2)$, dissipation drives the domain wall velocity $\dot{x}_0$ to zero. In contrast, the spin waves which lead to the dissipation function (55) will not tend to lower the domain velocity $\dot{x}_0$ but instead drive the relative velocity $\dot{q}=|\dot{x}_0-u|$ to zero. In other words, the dissipation through spin-wave emission acts to drive the system toward the state $\dot{x}_0=u$, in contrast to pure Gilbert damping for which the domain wall velocity is driven toward zero.

The equality between α and β restores Galilean invariance, which has previously been argued by Barnes and Maekawa.[23] In our theory, this invariance is restored because the dissipation channel, in this case the magnons, also "flows" with the effective drift velocity u through the Doppler effect. As such, the dissipation of the domain wall motion through spin-wave interactions leads to a Landau-type damping in which the motion tends toward $\dot{x}_0=u$. We contend, therefore, that *any* dissipation channel that drifts at the same velocity as the spin current u should lead to a similar dissipation channel, for which we can write symbolically as $\alpha_{\mathrm{drift}}=\beta_{\mathrm{drift}}$. In the context of the analogy to Landau damping, the charged particles of the plasma resemble the domain wall spins here and the field phase velocity is analogous to the spin-current velocity u. We do not wish to stress this analogy further as the details concerning the wave-particle interaction are quite different from the interactions between

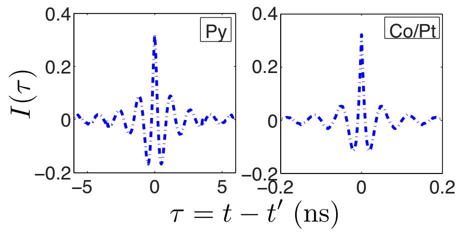

FIG. 5. (Color online) (a) Correlation function $I(t-t')$ between the stochastic torque $T_{\mathrm{stoc}}(t)$ at two different times t and t' for a wire of Permalloy (Ref. 18) with $K_\perp/K_u=57$ and $\hbar/K_u=1.88$ ns. (b) $I(t-t')$ for a wire of Co/Pt (Ref. 51) with $K_\perp/K_u=0.73$ and $\hbar/K_u=0.013$ ns.

the domain wall and magnons in our magnetic system.

Of course, extrinsic pinning centers such as magnetic impurities break the translational invariance of the wire, so dissipation of the domain wall motion through coupling to such impurities would not lead to a Galilean-invariant solution $\dot{x}_0=u$. Nevertheless, while we do not expect α and β to be equal in general, we have identified an important dissipation channel, i.e., spin-wave emission, that would tend to restore this translational symmetry. This is one of the key results of this paper.

C. Stochastic field

The force F_{stoc} (41) and the torque T_{stoc} (44), which accompany the dissipation by the magnons, lead to a domain wall motion that is stochastic. As $\langle\hat{\phi}_k\rangle=0$ and $\langle\hat{\theta}_k\rangle=0$, the average values of $F_{\mathrm{stoc}}(t)$ and $T_{\mathrm{stoc}}(t)$ vanish identically but their correlations are finite. Assuming the wire to be very large compared to the domain wall width λ, the autocorrelation of the force F_{stoc} is found to be

$$\langle F_{\mathrm{stoc}}(t)F_{\mathrm{stoc}}(t')\rangle = \dot{p}(t)\dot{p}(t')\left(\frac{a}{2\pi\lambda}\right)^3\frac{2k_BT}{K_u}I(t-t') + p(t)p(t')\times\left(\frac{K_u}{\hbar}\right)^2\left(\frac{a}{2\pi\lambda}\right)^3\frac{2k_BT}{K_u}J(t-t')$$

with

$$I(t-t') = \int d^3k\,\lambda^3(\eta_k^\phi)^2\left(\frac{\pi}{2}\right)^2\mathrm{sech}^2\left(\frac{k_x\lambda\pi}{2}\right)\frac{\cos\Omega_k(t-t')}{\omega_k^{3/2}(\omega_k+\kappa)^{1/2}} \tag{62}$$

and

$$J(t-t') = \int d^3k\,\lambda^3(\eta_k^\phi)^2\left(\frac{\pi}{2}\right)^2\mathrm{sech}^2\left(\frac{k_x\lambda\pi}{2}\right)\times\sqrt{\frac{\omega_k+\kappa}{\omega_k}}\cos\Omega_k(t-t'). \tag{63}$$

$I(t-t')$ is presented in Fig. 5 with the anisotropy constants K_u and $K_\perp$ taken to be those for Permalloy[18] and Co/Pt.[51] It is seen that $I(t-t')$ cancels out very quickly when $|t-t'|$ becomes larger than 1 ns. The function $J(t-t')$ behaves very similarly to $I(t-t')$ and weakens very quickly when the time difference $|t-t'|$ increases. We therefore conclude that the stochastic forces $F_{\mathrm{stoc}}(t)$ and $F_{\mathrm{stoc}}(t')$ are not correlated with each other at the characteristic time scale associated with domain wall motion, which usually is longer than the nanosecond.[52] For instance, the time scale of a domain wall at a typical velocity of 1 m/s in a Permalloy wire corresponds to $\lambda/(1\ \mathrm{ms}^{-1})\sim 50$ ns.

Analogously, the autocorrelation of the torque $T_{\mathrm{stoc}}(t)$ is found to be

$$\langle T_{\mathrm{stoc}}(t)T_{\mathrm{stoc}}(t')\rangle = \frac{p(t)}{m}\frac{p(t')}{m}\left(\frac{a}{2\pi\lambda}\right)^3\frac{2k_BT}{K_u}I(t-t')$$

and the intercorrelation between the force $F_{\mathrm{stoc}}(t)$ and the torque $T_{\mathrm{stoc}}(t')$ is found to be

$$\langle F_{\mathrm{stoc}}(t)T_{\mathrm{stoc}}(t')\rangle = \dot{p}(t)\frac{p(t')}{m}\left(\frac{a}{2\pi\lambda}\right)^3\frac{2k_BT}{K_u}I(t-t').$$

These correlations involve the same function $I(t-t')$ as before Eq. (62). From this study on the spin-wave fluctuations, we conclude that the stochastic force F_{stoc} and the stochastic torque T_{stoc} behave as a multiplicative white noise on the domain wall.

V. REDUCTION IN DOMAIN WALL WIDTH

External forces, such as a magnetic field or an electrical current, lead to deformations of the domain wall profile. An important consequence of such deformations is the generation of stray magnetic dipolar charges in the domain wall, which collectively build up the kinetic energy of the domain wall. Indeed, the kinetic energy $K_\perp\lambda^2p^2/2N_{\mathrm{dw}}S^2$ depends on the transverse anisotropy $K_\perp$, which for planar anisotropy materials, such as Permalloy, results from dipolar charges generated at the film surfaces that is associated with magnetization motion out of the film plane. Such wall deformation arises when the domain wall moves, for example, in the viscous regime in which the wall undergoes streaming motion at constant velocity under an applied external magnetic field.[53] In this section, we show that the deformation due to wall motion is not restricted to dynamics driven by a magnetic field but also appears in a similar manner under applied electrical currents.

The domain wall deformation can be described in terms of the modes $c_{\mathrm{loc}}(t)$, $c_k(t)$, and $c_k^\dagger(t)$. In Sec. III A the mode $c_{\mathrm{loc}}(t)$ has been interpreted as the kinetic energy of the domain wall. In the present section, we show that the modes $c_k^{\mathrm{def}}(t)$ and $c_k^{\dagger\mathrm{def}}(t)$ can be interpreted as a reduction in the domain wall width, which subsequently appears as a change in the domain wall mass.

Departing from Lagrangian (34) and considering the Euler-Lagrange equation with respect to $c_k^\dagger$, we obtain

$$-\frac{S}{2i}\dot{c}_k = -\left(\hbar\Omega_k - S(u-\dot{x}_0)\frac{k_x}{2}\right)c_k - (u-\dot{x}_0)v_kP + (u-\dot{x}_0)S\sum_{k\neq m}[iv_{-k,m}(c_m+c_{-m}^\dagger) - iv_{m,-k}(c_m-c_{-m}^\dagger)]. \tag{64}$$

The mode $c_{\boldsymbol{k}}$ has two components $c_{\boldsymbol{k}}^{\text{th}}$ and $c_{\boldsymbol{k}}^{\text{def}}$. The fast component $c_{\boldsymbol{k}}^{\text{th}}(t)$ (e.g., $\sim$4 GHz in Permalloy) represents the thermal excitations, whereas the slow component $c_{\boldsymbol{k}}^{\text{def}}$ (e.g., $\sim$20 MHz for $dx_0/dt=1$ m/s in Permalloy) represents wall deformation. As the differential equation (64) is linear, the components $c_{\boldsymbol{k}}^{\text{th}}$ and $c_{\boldsymbol{k}}^{\text{def}}$ can be calculated independently. The slow component $c_{\boldsymbol{k}}^{\text{def}}$ is almost static $\dot{c}_{\boldsymbol{k}}^{\text{def}}=0$ and corresponds to the "static" particular solution of Eq. (64). In contrast the fast component of $c_{\boldsymbol{k}}$ has no static part and is a homogeneous solution of Eq. (64).

Let us calculate the deformation mode $c_{\boldsymbol{k}}^{\text{def}}$ from Eq. (64) by approximating $\dot{c}_{\boldsymbol{k}}^{\text{def}}=0$. It is useful to consider the Taylor expansion of $c_{\boldsymbol{k}}^{\text{def}}$ with respect to the relative velocity $u-\dot{x}_0$,

$$c_{\boldsymbol{k}}^{\text{def}} = \sum_{n\geq 1} g_n(u-\dot{x}_0)^n. \tag{65}$$

Since the domain wall is much slower than the propagating spin waves $(u-\dot{x}_0)/\lambda \ll \Omega_{\boldsymbol{k}}$, it is sufficient to keep only the first order in the Taylor expansion (65),

$$c_{\boldsymbol{k}}^{\text{def}} \approx g_1(u-\dot{x}_0). \tag{66}$$

One readily obtains

$$c_{\boldsymbol{k}}^{\text{def}} \approx -\frac{(u-\dot{x}_0)v_{\boldsymbol{k}}}{\hbar\Omega_{\boldsymbol{k}}}p. \tag{67}$$

Therefore the deviation $\delta\phi^{\text{def}}$ in the spherical angle ϕ due to deformation is

$$\delta\phi^{\text{def}} = \sum_{\boldsymbol{k}} (c_{\boldsymbol{k}}^{\text{def}} + c_{-\boldsymbol{k}}^{\text{def}*})\nu_{\boldsymbol{k}}^{\phi}\xi_{\boldsymbol{k}}, \tag{68}$$

$$\delta\phi^{\text{def}} = -2\sum_{\boldsymbol{k}} \frac{(u-\dot{x}_0)p}{\hbar\Omega_{\boldsymbol{k}}}(\nu_{\boldsymbol{k}}^{\phi})^2\frac{\pi}{2}\text{sech}\left(\frac{k_x\lambda\pi}{2}\right)\frac{1}{\sqrt{\omega_{\boldsymbol{k}}N}}\xi_{\boldsymbol{k}}. \tag{69}$$

By letting $(\nu_{\boldsymbol{k}}^{\phi})^2=(1/4)\sqrt{(\omega_{\boldsymbol{k}}+\kappa)/\omega_{\boldsymbol{k}}}$ and $p=(\dot{x}_0-u)m$, one finds

$$\delta\phi^{\text{def}} = \frac{\pi}{4}\sum_{\boldsymbol{k}} \frac{S^2(u-\dot{x}_0)^2}{K_uK_\perp\lambda}\frac{1}{\omega_{\boldsymbol{k}}}\frac{N_\perp}{a}\text{sech}\left(\frac{k_x\lambda\pi}{2}\right)\frac{1}{\sqrt{\omega_{\boldsymbol{k}}N}}\xi_{\boldsymbol{k}}. \tag{70}$$

Let us now calculate the deformation angle $\delta\phi^{\text{def}}$ corresponding to a change $\delta\lambda$ in the domain wall width. Its comparison to Eq. (70) will then allow us to estimate $\delta\lambda$. The ϕ angle describing the magnetization inside a Bloch domain wall is

$$\phi_0(x) = \pi + \cos^{-1}\left[\tanh\left(\frac{x-x_0}{\lambda_0}\right)\right]. \tag{71}$$

If the domain wall width is modified by the spin current by an amount $\delta\lambda(u)$, the actual magnetization profile ϕ will deviate from the equilibrium Bloch profile ϕ_0 by

$$\phi = \phi_0 - \frac{x-x_0}{\lambda^2}\sqrt{1+\tanh^2\left(\frac{x-x_0}{\lambda_0}\right)}\delta\lambda. \tag{72}$$

The deformation angle $\delta\phi^{\text{def}}(\boldsymbol{r}-\boldsymbol{r}_0)=\phi(\boldsymbol{r})-\phi_0(\boldsymbol{r})$ can be expanded on the wave functions $\xi_k(\boldsymbol{r}-\boldsymbol{r}_0)$ as

$$\delta\phi(\boldsymbol{r}-\boldsymbol{r}_0) = \sum_{\boldsymbol{k}} p_{\boldsymbol{k}}\xi_{\boldsymbol{k}}(\boldsymbol{r}-\boldsymbol{r}_0), \tag{73}$$

where

$$p_{\boldsymbol{k}} = \int \frac{d^3r}{a^3}\delta\phi(\boldsymbol{r})\xi_{-\boldsymbol{k}}(\boldsymbol{r}),$$

$$= -\int \frac{d^3r}{a^3}\frac{x-x_0}{\lambda^2}\sqrt{1+\tanh^2\left(\frac{x-x_0}{\lambda_0}\right)}\delta\lambda\,\xi_{-\boldsymbol{k}}(\boldsymbol{r}).$$

Only the real part of the wave function $\xi_{-\boldsymbol{k}}(\boldsymbol{r})$ will contribute to $p_{\boldsymbol{k}}$,

$$\text{Re}[\xi_{-\boldsymbol{k}}(\boldsymbol{r})] = \frac{1}{\sqrt{N\omega_{\boldsymbol{k}}}}\left[\cos[k_x(x-x_0)]\tanh\left(\frac{x-x_0}{\lambda}\right) + k_x\lambda\sin k_x(x-x_0)\right]\cos(\boldsymbol{k}_\perp\cdot\boldsymbol{r}).$$

Accordingly,

$$p_{\boldsymbol{k}} = -\frac{\delta\lambda}{a}\frac{N_\perp\pi}{\sqrt{N\omega_{\boldsymbol{k}}}}\text{sech}\left(\frac{k_x\lambda\pi}{2}\right). \tag{74}$$

Hence the change $\delta\lambda$ in the domain wall width corresponds to the deformation angle

$$\delta\phi^{\text{def}} = -\sum_{\boldsymbol{k}}\frac{\delta\lambda}{a}\frac{N_\perp\pi}{\sqrt{N\omega_{\boldsymbol{k}}}}\text{sech}\left(\frac{k_x\lambda\pi}{2}\right)\xi_{\boldsymbol{k}}. \tag{75}$$

By comparing Eqs. (70) and (75) and by approximating $\omega_{\boldsymbol{k}} \approx 1$, we finally obtain

$$\frac{\delta\lambda}{\lambda} = -\frac{1}{4}\frac{S^2(u-\dot{x}_0)^2}{K_uK_\perp\lambda^2}. \tag{76}$$

Equation (76) shows that the domain wall shrinks under the effect of spin-transfer torque. According to Eq. (40), this reduction in width leads to a force F_{def} that is nonlinear with respect to the relative velocity $u-\dot{x}_0$,

$$F_{\text{def}} = -c\frac{d}{dt}\left[p\frac{S^2(u-\dot{x}_0)^2}{K_uK_\perp\lambda^2}\right] \tag{77}$$

with $c=(\pi/32)\int_{-\infty}^{+\infty}dk_x\,\text{sech}^2(\pi k_x/2)/(1+k_x^2)\sim 0.1$. In the case of Permalloy nanowires, $K_\perp$ represents the demagnetizing energy and cancels with the factor S^2 in the numerator in Eq. (77). Therefore F_{def} does not depend strongly on the transverse anisotropy $K_\perp$.

The deformation force F_{def} can be reinterpreted as a modification of the kinetic momentum and of the mass of the domain wall. As the domain wall deforms, its kinetic momentum becomes

$$p + \delta p = p\left(1+\frac{3cS^2(u-\dot{x}_0)^2}{K_\perp A}\right), \tag{78}$$

which corresponds to a change in mass,

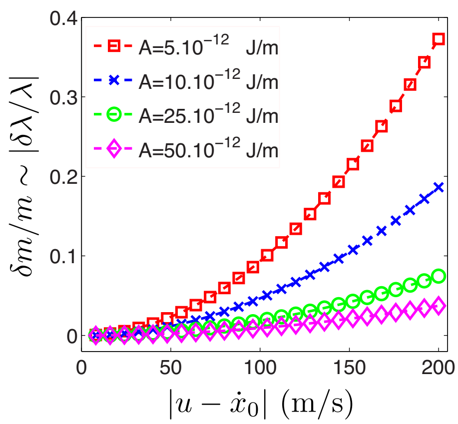

FIG. 6. (Color online) Modification of the domain wall mass m and of the domain wall width λ as a function of the relative velocity $|u-\dot{x}_0|$ and the exchange stiffness constant A.

$$\frac{\delta m}{m} = \frac{6c}{A\mu_0}(u-\dot{x}_0)^2\left(\frac{\hbar}{g\mu_B}\right)^2. \tag{79}$$

The modifications of the domain wall mass and of the domain wall width, for which the relative changes are almost identical in magnitude but differ in sign, $\delta m/m \sim -\delta\lambda/\lambda$, are represented in Fig. 6 as a function of the relative velocity $|u-\dot{x}_0|$ and for different values of the exchange stiffness constant A.

VI. TEMPERATURE DEPENDENCE OF THE DOMAIN WALL MASS

The flow of an electrical current through a ferromagnetic wire modifies the spectrum of the magnons according to Eq. (33), which thereby shifts their average kinetic momentum to a finite value $\bar{\boldsymbol{k}}=\Sigma_{\boldsymbol{k}}\boldsymbol{k}n_B(\boldsymbol{k})/\Sigma_{\boldsymbol{k}}n_B(\boldsymbol{k})$. The applied electrical current therefore leads to a magnon current. Inside a domain wall, magnons behave slightly differently because of the singular coupling potential $-S(u-\dot{x}_0)\Sigma_{\boldsymbol{km}}v_{\boldsymbol{km}}\hat{\theta}_{\boldsymbol{k}}\hat{\phi}_{\boldsymbol{m}}$ (34). This potential may be interpreted as a modification of the domain wall energy due to its interaction with the spin-wave environment. This section is devoted to this coupling potential, whereby we investigate its consequences on the domain wall dynamics. Specifically, this potential will be shown to contribute to the forces F_j (39) and F_H (38) on the domain wall.

The relationship between the forces F_j and F_H and the statistics of the magnons will be established in Sec. VI A. The force F_j arising from the dc component of u will be carried out in Sec. VI B and the force F_H arising from the ac component $u(t)$ will then be calculated in Sec. VI C. Both these forces will be reinterpreted as a modification of the effective mass of the domain wall. The domain wall mass will in turn become sensitive to the actual temperature of the ferromagnetic wire.

A. Forces F_j and F_H

While the force F_j depends on the correlation $\langle\delta\phi\delta\theta\rangle$ between $\delta\theta$ and $\delta\phi$, the force F_H depends on the autocorrelation $\langle\delta\theta^2\rangle$ and $\langle\delta\phi^2\rangle$. These correlations, which are finite under the effect of an electrical current, can be calculated with perturbation or linear-response theory. In order to keep the notation as simple as possible, we will employ a 2×2 matrix representation for the response functions; details concerning this notation can be found in the Appendix.

The correlations between the angular deviations $\delta\phi$ and $\delta\theta$ are well described by the lesser magnon Green's function $\mathcal{D}^<(\boldsymbol{r},t,\boldsymbol{r}',t')$ defined as

$$i\hbar\mathcal{D}^< = \begin{pmatrix} \langle\delta\theta(\boldsymbol{r}',t')\delta\theta(\boldsymbol{r},t)\rangle & \langle\delta\phi(\boldsymbol{r}',t')\delta\theta(\boldsymbol{r},t)\rangle \\ \langle\delta\theta(\boldsymbol{r}',t')\delta\phi(\boldsymbol{r},t)\rangle & \langle\delta\phi(\boldsymbol{r}',t')\delta\phi(\boldsymbol{r},t)\rangle \end{pmatrix}.$$

The representation of the lesser function in the momentum space $\mathcal{D}^<(\boldsymbol{k},t,\boldsymbol{k}',t')$ is obtained by expanding $\delta\theta$ and $\delta\phi$ on the wave functions $\xi_{\boldsymbol{k}}$, according to Eqs. (18) and (19). The representation in momentum space is quite useful as it reveals the relationship between the angular deviations $\delta\phi$, $\delta\theta$ and the statistics of the magnons.

The force F_H depends on the diagonal component of the lesser function $\mathcal{D}^<_{\parallel}=\mathcal{D}^<_{\theta\theta}=\mathcal{D}^<_{\phi\phi}$,

$$F_H = \sum_{\boldsymbol{kq}} f^H(\boldsymbol{k},\boldsymbol{q})i\hbar\mathcal{D}^<_{\parallel}(\boldsymbol{k}_-,t,\boldsymbol{k}_+,t), \tag{80}$$

where $\boldsymbol{k}_-=\boldsymbol{k}-\boldsymbol{q}/2$ and $\boldsymbol{k}+\boldsymbol{q}/2$ and

$$f^H(\boldsymbol{k},\boldsymbol{q}) = -\frac{K_u}{3\lambda}\frac{2\pi}{L}\frac{1}{i}\text{sech}\left(\frac{\pi\lambda q_x}{2}\right)\lambda q_x \times \frac{1+3(k_x\lambda)^2+\dfrac{(q_x\lambda)^2}{4}}{\sqrt{\omega_{\boldsymbol{k}_-}\omega_{\boldsymbol{k}_+}}}(\nu^\theta_{\boldsymbol{k}_-}\nu^\theta_{\boldsymbol{k}_+}+\nu^\phi_{\boldsymbol{k}_-}\nu^\phi_{\boldsymbol{k}_+}). \tag{81}$$

The force F_j depends on the off-diagonal component of the lesser function $\mathcal{D}^<_{\text{off}}=\mathcal{D}^<_{\phi\theta}$,

$$F_j = \sum_{\boldsymbol{kq}} f^j(\boldsymbol{k},\boldsymbol{q})\frac{d}{dt}i\hbar\mathcal{D}^<_{\text{off}}(\boldsymbol{k}_-,t,\boldsymbol{k}_+,t) \tag{82}$$

with

$$f^j(\boldsymbol{k},\boldsymbol{q}) = -Sv_{-\boldsymbol{k}_-,\boldsymbol{k}_+}. \tag{83}$$

We recall that the coefficient $v_{-\boldsymbol{k}_-,\boldsymbol{k}_+}$ is defined in Eq. (32) and that the coefficients $\nu^\theta_{\boldsymbol{k}}$ and $\nu^\phi_{\boldsymbol{k}}$ are given in Sec. III B.

As $f^H(\boldsymbol{k},\boldsymbol{q})$ is an odd function in $\boldsymbol{q}$ and an even function in $\boldsymbol{k}$, the force F_H will be finite if $\mathcal{D}^<_{\parallel}(\boldsymbol{k}_-,t,\boldsymbol{k}_+,t)$ is odd in $\boldsymbol{q}$ and even in $\boldsymbol{k}$. It will be shown in Sec. VI C that the nonadiabatic response of the spin waves to a dynamical spin current $u(t)$ gives rise to such a lesser Green's function $\mathcal{D}^<$. Furthermore, as $f^j(\boldsymbol{k},\boldsymbol{q})$ is odd in $\boldsymbol{k}$ and even in $\boldsymbol{q}$, F_j will be finite if $\mathcal{D}^<_{\text{off}}(\boldsymbol{k}_-,t,\boldsymbol{k}_+,t)$ is odd in $\boldsymbol{k}$ and even in $\boldsymbol{q}$. We will see in Sec. VI B that this is the case for the adiabatic response of the spin waves to a dc spin current u.

B. Adiabatic response

The process in Fig. 7 depicts the adiabatic transfer of kinetic momentum between the propagating magnons and the domain wall, which arises from the coupling potential $\mathcal{V}$

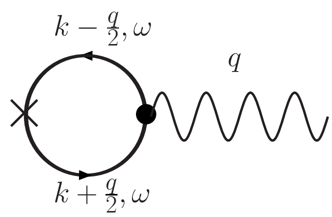

FIG. 7. Representation of the force F_j created by the spin waves when the relative velocity $u-\dot{x}_0$ is finite and the motion is adiabatic. The wavy line represents the magnon q exchanged with the domain wall, the vertex $\times$ arises from the force F_j and the vertex ● represents the potential $\mathcal{V}$.

(35). Due to adiabaticity, the energies of the incoming magnon $\boldsymbol{k}-\frac{\boldsymbol{q}}{2}$ and the outgoing magnon $\boldsymbol{k}+\frac{\boldsymbol{q}}{2}$ are identical.

The statistics of the magnons perturbed by the coupling potential $\mathcal{V}$ can be investigated by expanding the contour-ordered magnon propagator $\mathcal{D}(\boldsymbol{k}-\frac{\boldsymbol{q}}{2},\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\tau,\tau')$ up to the first order in $\mathcal{V}$,

$$\mathcal{D}^{(1)}(\boldsymbol{k}_-,\tau,\boldsymbol{k}_+,\tau') = -S(u-\dot{x}_0)\int_C d\tau_1\, iv_{-\boldsymbol{k}_-\boldsymbol{k}_+}(\tau_1) \times\mathcal{D}^{(0)}(\boldsymbol{k}_-,\tau-\tau_1)\sigma_y\mathcal{D}^{(0)}(\boldsymbol{k}_+,\tau_1-\tau'). \tag{84}$$

In Eq. (84) the superscript (0) represents equilibrium ($\dot{x}_0 = u$), whereas the superscript (1) represents first-order perturbation theory. The lesser Green's function $\mathcal{D}(\boldsymbol{k}_-,\boldsymbol{k}_+,t-t')$ can be obtained from the time-ordered Green's function with the Langreth formula $(\mathcal{D}\mathcal{D})^< = \mathcal{D}^r\mathcal{D}^< + \mathcal{D}^<\mathcal{D}^a$ and the fluctuation-dissipation theorem,

$$\mathcal{D}^{(0)<}(\boldsymbol{k},\omega) = n_B(\omega)[\mathcal{D}^{(0)r}(\boldsymbol{k},\omega) - \mathcal{D}^{(0)a}(\boldsymbol{k},\omega)]. \tag{85}$$

By denoting Im(z) the imaginary part of z and

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$

the lesser Green's function is found in terms of the retarded [Eq. (A7)] and advanced [Eq. (A8)] propagators as

$$\mathcal{D}^{(1)<}(\boldsymbol{k}_-,\boldsymbol{k}_+,\omega) = 4S(u-\dot{x}_0)v_{-\boldsymbol{k}_-\boldsymbol{k}_+}n_B(\omega) \times\{I\,\mathrm{Im}[g_+^{(0)r}(\boldsymbol{k}_-,\omega)g_+^{(0)r}(\boldsymbol{k}_+,\omega) - g_-^{(0)r}(\boldsymbol{k}_-,\omega)g_-^{(0)r}(\boldsymbol{k}_+,\omega)] + \sigma_y\,\mathrm{Im}[g_+^{(0)r}(\boldsymbol{k}_-,\omega)g_+^{(0)r}(\boldsymbol{k}_+,\omega) + g_-^{(0)r}(\boldsymbol{k}_-,\omega)g_-^{(0)r}(\boldsymbol{k}_+,\omega)]\}. \tag{86}$$

From this equation, we can infer that $\mathcal{D}_{\parallel}^{(1)<}(\boldsymbol{k}_-,t,\boldsymbol{k}_+,t)=0$. Thus by inspection the adiabatic response of the spin waves to $(u-\dot{x}_0)$ does not contribute to the force F_H (80). In contrast, the off-diagonal component $\mathcal{D}_{\mathrm{off}}^{(1)<}(\boldsymbol{k}_-,\boldsymbol{k}_+,t)$ of the lesser Green's function is finite when the domain wall velocity differs from the velocity of the spin current $\dot{x}_0 \neq u$. It is found to be

$$i\hbar\mathcal{D}_{\mathrm{off}}^{(1)<}(\boldsymbol{k}_-,\boldsymbol{k}_+,t) = -8\pi S(u-\dot{x}_0)v_{-\boldsymbol{k}_-\boldsymbol{k}_+}\left[\frac{\partial n_B(\epsilon_k)}{\partial\epsilon}\right]$$

and leads to a force F_j (82)

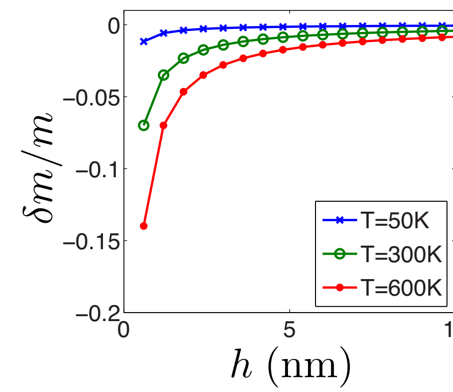

FIG. 8. (Color online) Modification of the domain wall mass $\delta m/m$ as a function of the wire height h, for different temperatures T, assuming $L_y=100$ nm and $\kappa=57$.

$$F_j = -\frac{dp}{dt}\frac{8\pi K_\perp\lambda^2}{N_{\mathrm{dw}}}\sum_{\boldsymbol{k},\boldsymbol{q}}\frac{\partial n_B(\epsilon_k)}{\partial\epsilon}(v_{-\boldsymbol{k}_-\boldsymbol{k}_+})^2. \tag{87}$$

Being proportional to dp/dt, the force F_j may be reinterpreted as a modification of the domain wall mass. In thin films in which the magnetization can be approximated to be uniform over the thickness of the film, the change in the domain wall mass is found to be

$$\frac{\delta m}{m} = \frac{k_B T a^2}{A}\frac{2\pi}{N_z}\xi(\kappa,L_y), \tag{88}$$

where N_z represents the number of atomic layers that constitute the film thickness, a is the lattice constant, L_y represents the width of the wire, and $\xi(\kappa,L_y)$ is a function of L_y and the anisotropies $\kappa=K_\perp/K_u$. The change in the mass $\delta m/m$ is presented in Fig. 8 as a function of the wire thickness $h = N_z a$ and for different values of the temperature. In a Permalloy wire with a cross section 2×100 nm^2, the mass of the domain wall varies by $\sim$5% between $T=0$ K and $T = 600$ K. This variation of the domain wall mass could be much larger near the Curie temperature $T_c \sim 750$ K.

C. Nonadiabatic response

The process in Fig. 9 shows the nonadiabatic transfer of kinetic momentum between the propagating magnons and

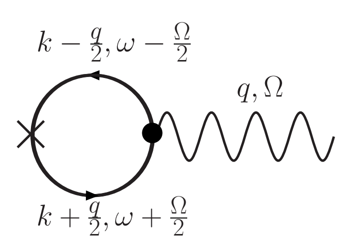

FIG. 9. Representation of the force F_H created by the spin waves when the excitation of the spin waves by the spin current is not adiabatic. The wavy line represents the momentum $\boldsymbol{q}$ exchanged with the domain wall and the energy Ω transferred by the spin current; the vertex $\times$ arises from the force F_H and the vertex ● represents the mixing potential $\mathcal{V}$.

the domain wall, which arises from the coupling potential $\mathcal{V}$ (35) when the domain wall is accelerating. Through this process, the energy of magnons is not conserved.

In the following, the domain wall motion is assumed to be harmonic $u(t) \propto e^{i\Omega t}$. This assumption will allow us to calculate the linear response of the spin waves to the domain wall acceleration in a more tractable way. The frequency Ω of the domain wall acceleration (e.g., $\sim$10 MHz) is much lower than the spin-wave eigenfrequencies $\Omega_{\boldsymbol{k}}$ ($\sim$5 GHz) but still induces an overlap between the different spin-wave modes. As the actual overlap between the spin waves depends on their spectral linewidth, the lifetime $\tau_{\boldsymbol{k}}$ of the magnons is a key physical quantity in this problem. According to the phenomenological Gilbert damping, each magnon $\boldsymbol{k}$ has a lifetime $\tau_{\boldsymbol{k}}$ inversely proportional to its eigenfrequency and to the Gilbert coefficient $\tau_{\boldsymbol{k}}^{-1}=\alpha\Omega_{\boldsymbol{k}}$. The linewidth of each spin-wave mode is therefore about $\Delta f=0.01\times 4$ GHz $\sim$40 MHz. As the wire length L_x is generally much larger than the domain wall width λ, the energy gap between the peaks of two consecutive spin-wave modes is on the order of $\sqrt{K_u K_\perp}(2\pi\lambda/L_x)^2/2\sim 5$ MHz.

For the sake of determining the linear response of the spin waves to the domain wall acceleration, we use the Dyson equation (84) as a starting point. As $\Omega/\Omega_{\boldsymbol{k}}\ll 1$, we linearize $\mathcal{D}^{(1)<}$ with respect to Ω,

$$\mathcal{D}^{(1)<}\left(\boldsymbol{k}-\frac{\boldsymbol{q}}{2},\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\omega,\omega+\Omega\right) = -iu_{-\boldsymbol{k}+\boldsymbol{q}/2,\boldsymbol{k}+\boldsymbol{q}/2}(-\Omega)\left[\frac{\partial n_B(\omega)}{\partial \epsilon}\right] \times\hbar\Omega\mathcal{D}^{(0)r}\left(\boldsymbol{k}-\frac{\boldsymbol{q}}{2},\omega\right)\sigma_y\mathcal{D}^{(0)a}\left(\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\omega\right). \quad (89)$$

The diagonal component of the lesser magnon Green's function is then obtained as

$$\mathcal{D}_{\parallel}^{(1)<}(\boldsymbol{k}_-,t,\boldsymbol{k}_+,t)=\frac{\alpha S}{m}\frac{dp}{dt}k_B T\Phi(k_x,q_x) \quad (90)$$

with

$$\Phi(k_x,q_x) = \frac{\lambda}{L}k_x q_x \lambda^2 \operatorname{csch}\left(\frac{\pi\lambda q_x}{2}\right)4\pi\frac{\nu_{k_-}^{\theta}\nu_{k_+}^{\phi}}{\sqrt{\omega_{k_-}\omega_{k_+}}}\int\frac{d\omega}{\omega} \times\frac{\epsilon_{k_+}-\epsilon_{k_-}}{[(\hbar\omega-\epsilon_{k_-})^2+(\alpha\epsilon_{k_-})^2][(\hbar\omega-\epsilon_{k_+})^2+(\alpha\epsilon_{k_+})^2]}.$$

The force F_H due to this nonadiabatic interaction is derived by carrying over the magnon Green's function (90) to the definition of the force (80). Because it is proportional to dp/dt, the force F_H can be reinterpreted as a modification of the domain wall mass. The contributions of the transverse spin-wave modes n_y to $\delta m/m$ are shown in Fig. 10 for different values of the temperature. The latter figure indicates that the change in the domain wall mass is in general less than 1/1000. This implies that the nonadiabatic response of the spin waves to the domain wall acceleration does not affect domain wall motion significantly. This is in contrast with the adiabatic response of the spin waves (Sec. VI B).

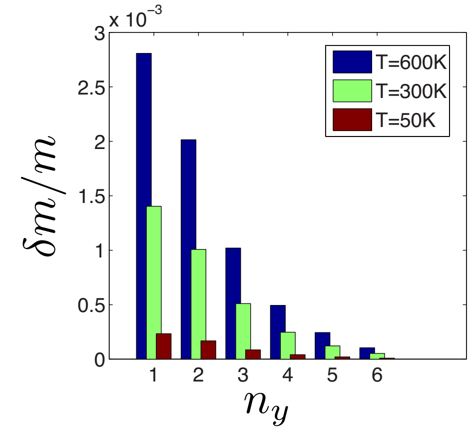

FIG. 10. (Color online) Contribution of the transverse spin-wave modes n_y to the change in the domain wall mass $\delta m/m$. L_y =400 nm, α=0.01, and $\kappa=K_\perp/K_u$=57.

VII. DISCUSSION AND CONCLUSION

We have developed a theory of current-driven domain wall dynamics in magnetic wires which takes into account interactions between the domain wall and propagating spin waves. In general, a spin current u traversing the magnetic medium will lead to an interaction between the domain wall and the propagating spin-wave modes. This interaction is proportional to the kinetic energy of the domain wall, $m(u-\dot{x}_0)^2/2$. As this kinetic energy is governed by the difference between the spin current u and the domain wall velocity $\dot{x}_0$, a domain wall moving at the spin-current velocity will not interact with the spin waves. This coupling between the domain wall and the propagating magnons is shown to have two main effects on the domain wall dynamics: (1) damping of the domain wall motion and (2) renormalization of the domain wall effective mass. Dissipation associated with the emission of magnons is found to drive the wall velocity toward the spin-current velocity u, such that the relative velocity $\dot{x}_0-u$ is minimized. This dissipation process is analogous to Landau dissipation in plasmas. The renormalization of the domain wall effective mass has two origins: the reduction in the domain wall width and the renormalization of the domain wall energy by thermal magnons.

In the presence of Gilbert damping and additional spin-wave interactions, our extended one-dimensional model of Bloch wall dynamics is given by

$$\dot{p}+\frac{2K_\perp m^*}{S}\left(\alpha+\frac{\eta p^2}{N_{\mathrm{dw}}K_\perp m^*}\right)\dot{x}_0=\frac{2K_\perp m^*}{S}\left(\beta+\frac{\eta p^2}{N_{\mathrm{dw}}K_\perp m^*}\right)u \quad (91)$$

and

$$\frac{p}{m^*}=\dot{x}_0-u-\frac{\alpha S}{2m^* K_\perp}p. \quad (92)$$

The effective mass m^* takes into account the deformation of the domain wall and its coupling to the magnons. As such, it depends on the actual temperature of the wire. The emission of magnons, which is parametrized by the dimensionless coefficient η, is important if the coefficient η is close to 1.

The equation of motion (91) shows explicitly how magnon emission, which contributes to the dissipation of the domain wall motion, also contributes with the same magnitude to the nonadiabatic β term. In the limit in which the only dissipation channel for domain wall motion is through the interaction with spin waves, we find that Galilean invariance is restored and $\alpha=\beta$. This result can be generalized for dissipation involving any subsystem to which the domain wall is coupled that drifts at the same velocity as the spin current u. Our spin-wave result provides a natural explanation as to why the magnitudes of α and β are often found to be very similar in experiment. However, we expect Galilean invariance to be broken in realistic systems due to the presence of inhomogeneities, magnetic impurities, and ultimately the nature of the underlying atomic crystal. As such, we do not expect α and β to be strictly equal in general. Nevertheless, the key result here, whereby spin-wave emission in the presence of purely adiabatic torques gives rise to an apparent nonadiabatic term, highlights the importance of identifying the translational symmetry of relevant dissipation processes for any subsequent study of the nonadiabatic contributions to current-driven domain wall motion.

Results (91) and (92) may appear inconsistent in the sense that Gilbert damping is included for the domain wall and not for the spin waves. While we have not included damping explicitly for the spin-wave system, dissipation of magnetization from the spin-wave system toward other thermal baths (e.g., phonons and magnetic impurities) is implied in our treatment of domain wall damping. In the context of the Caldeira-Leggett approach, our method relies on drawing the analogy between the domain wall (spin-wave) system and the massive particle (harmonic oscillator) system. It is implicitly assumed that any energy transferred from the domain wall (massive particle) to the spin waves (thermal bath of oscillators) will be dissipated rapidly from the spin-wave system, such that there would be no transfer of energy back into the domain wall motion. As such, we have implicitly assumed a damping in the spin-wave system but we have not used Gilbert's model explicitly to that effect. For example, it is known that other higher-order magnon interaction processes, such as three- or four-magnon process,[30] lead to decoherence or dissipation of a spin-wave mode on time scales that are much faster than domain wall motion; such processes would contribute to the damping of the domain wall motion.

Equations (91) and (92) describe current-induced domain wall motion by including some mechanisms that depend on the temperature. Temperature enters domain wall dynamics through the domain wall mass m^* and also possibly through the emission coefficient η. According to Yamaguchi *et al.*,[35] the temperature inside ferromagnetic nanowires studied in experiment may approach the Curie temperature T_c as a result of Joule heating. According to our theory, such an increase in the temperature would appear as a renormalization of the domain wall mass. Our calculation suggests that a change in the domain wall mass of about 5% can occur for variations in temperature between $T=600$ K and $T=0$ K.

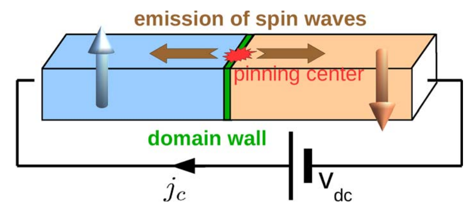

FIG. 11. (Color online) Proposed experiment for spin-wave emission. A dc power supply $V_{\rm dc}$ generates a charge current j_c, which in turn excites a domain wall pinned inside a wire. As the spin current u is finite but at the same time the domain wall is fixed, the relative velocity $u-\dot{x}_0$ is finite and spin waves are emitted.

This change becomes even more important close to the Curie temperature T_c. Laufenberg *et al.*[32] found a significant decrease in the spin-transfer efficiency, which is proportional to β, when the temperature was increased by few hundreds of kelvin above ambient temperatures. These authors have suggested emission of magnons to be responsible for this loss of efficiency. In the context of our theory, this could be explained by the decrease in magnetization that accompanies any increase in temperature, which would contribute to a larger β (and α) because of the reduction in $K_\perp$.

According to the present theory, current-driven domain walls might be viewed as the generators of spin waves. One experimental realization of such spin-wave generation is proposed in Fig. 11, where we show a domain wall pinned inside a wire that is traversed by a dc spin current. The action of the spin current, as we have discussed in some details above, acts to deform the domain wall and gives rise to a nonequilibrium magnetic configuration. As its velocity vanishes $\dot{x}_0=0$, its energy $m^*(u-\dot{x}_0)^2/2$ will become very large. A part of this energy will then be dissipated to the medium through the emission of magnons.

ACKNOWLEDGMENTS

Y.L.M. thanks D. Ravelosona for the important guidance and supervision of his work. He gratefully acknowledges very fruitful discussions with J. Shibata, H. Kohno, A. Takeuchi, K. Hosono, E. E. Fullerton, F. Piéchon, X. Waintal, R. Jalabert, A. Thiaville, T. Devolder, C. Chappert, V. Vlaminck, S. Kasai, and T. Ono. He also thanks Tokyo Metropolitan University for financial support during his stay in Minami Osawa and support from the CILOMAG contract of the French National Research Agency (ANR).

APPENDIX: PROPAGATORS

The magnon propagators allow the linear response of the spin waves to be derived in a very tractable way. The magnon propagators are defined by means of the magnon operators $\hat{\phi}_{\boldsymbol{k}}=\hat{c}_{\boldsymbol{k}}+\hat{c}^{\dagger}_{-\boldsymbol{k}}$ and $\hat{\theta}_{\boldsymbol{k}}=\frac{1}{i}(\hat{c}_{\boldsymbol{k}}-\hat{c}^{\dagger}_{-\boldsymbol{k}})$. Specifically the time-ordered magnon propagator $\mathcal{D}_{\alpha\beta}(\boldsymbol{k},\boldsymbol{k}',t,t')$ for $\alpha,\beta \in\{\theta,\phi\}$ is defined as

$$\mathcal{D}_{\alpha\beta}(\boldsymbol{k},\boldsymbol{k}',t,t')=-\frac{i}{\hbar}\langle\, |T\hat{\alpha}_{\boldsymbol{k}}(t)\hat{\beta}_{-\boldsymbol{k}'}(t')|\,\rangle \tag{A1}$$

with

$$T\hat{\alpha}_{\boldsymbol{k}}(t)\hat{\beta}_{-\boldsymbol{k}'}(t') = \begin{cases} \hat{\alpha}_{\boldsymbol{k}}(t)\hat{\beta}_{-\boldsymbol{k}'}(t') & \text{if } t > t' \\ \hat{\beta}_{-\boldsymbol{k}'}(t')\hat{\alpha}_{\boldsymbol{k}}(t) & \text{if } t < t' \end{cases}.$$

It is useful to let the times t and t' belong to a contour in the complex plane, which allows the Green's function (A1) to contain information on the nonequilibrium properties of the spin waves. The lesser and greater propagators are defined as

$$\mathcal{D}^{<}_{\alpha\beta}(\boldsymbol{k},\boldsymbol{k}',t,t') = -\frac{i}{\hbar}\langle\,|\hat{\beta}_{-\boldsymbol{k}'}(t')\hat{\alpha}_{\boldsymbol{k}}(t)|\,\rangle,$$

$$\mathcal{D}^{>}_{\alpha\beta}(\boldsymbol{k},\boldsymbol{k}',t,t') = -\frac{i}{\hbar}\langle\,|\hat{\alpha}_{\boldsymbol{k}}(t)\hat{\beta}_{-\boldsymbol{k}'}(t')|\,\rangle,$$

respectively, and the retarded and advanced propagators are

$$\mathcal{D}^{r}_{\alpha\beta}(t,t') = \theta(t-t')[\mathcal{D}^{>}_{\alpha\beta}(t,t') - \mathcal{D}^{<}_{\alpha\beta}(t,t')],$$

$$\mathcal{D}^{a}_{\alpha\beta}(t,t') = \theta(t'-t)[\mathcal{D}^{<}_{\alpha\beta}(t,t') - \mathcal{D}^{>}_{\alpha\beta}(t,t')],$$

respectively. To simplify the notation, we represent the various magnon propagators by means of 2×2 matrices in the form

$$\mathcal{D}(\boldsymbol{k},\boldsymbol{k}',\tau,\tau') = \begin{pmatrix} \mathcal{D}_{\theta,\theta}(\boldsymbol{k},\boldsymbol{k}',\tau,\tau') & \mathcal{D}_{\theta,\phi}(\boldsymbol{k},\boldsymbol{k}',\tau,\tau') \\ \mathcal{D}_{\phi,\theta}(\boldsymbol{k},\boldsymbol{k}',\tau,\tau') & \mathcal{D}_{\phi,\phi}(\boldsymbol{k},\boldsymbol{k}',\tau,\tau') \end{pmatrix}.$$

We use the interaction picture with respect to the unperturbed spin-wave Hamiltonian $\mathcal{H}_0=\Sigma_{\boldsymbol{k}}\hbar\Omega_{\boldsymbol{k}}c^{\dagger}_{\boldsymbol{k}}c_{\boldsymbol{k}}$. The creation and annihilation operators of the magnons have the time dependences $\hat{c}_{\boldsymbol{k}}(t)=\hat{c}_{\boldsymbol{k}}e^{-i\Omega_{\boldsymbol{k}}t}$ and $\hat{c}^{\dagger}_{\boldsymbol{k}}(t)=\hat{c}^{\dagger}_{\boldsymbol{k}}e^{i\Omega_{\boldsymbol{k}}t}$.

Before calculating the response of the spin waves to the coupling potential $\mathcal{V}$, the unperturbed propagators need to be determined. The free retarded and the free advanced magnon propagators are obtained as

$$\mathcal{D}^{(0)r}(\boldsymbol{k},\omega) = \left(\frac{I+\sigma_y}{\hbar\omega - \epsilon_{\boldsymbol{k}} + i0} - \frac{I-\sigma_y}{\hbar\omega + \epsilon_{\boldsymbol{k}} + i0}\right) \quad \text{(A2)}$$

and

$$\mathcal{D}^{(0)a}(\boldsymbol{k},\omega) = \left(\frac{I+\sigma_y}{\hbar\omega - \epsilon_{\boldsymbol{k}} - i0} - \frac{I-\sigma_y}{\hbar\omega + \epsilon_{\boldsymbol{k}} - i0}\right). \quad \text{(A3)}$$

The free lesser Green's function is given by the fluctuation-dissipation theorem

$$\mathcal{D}^{(0)<}(\boldsymbol{k},\omega) = n_B(\omega)[\mathcal{D}^{(0)r}(\boldsymbol{k},\omega) - \mathcal{D}^{(0)a}(\boldsymbol{k},\omega)]. \quad \text{(A4)}$$

It is useful to introduce the electronlike propagators $g^{(0)r}_{\pm}(\boldsymbol{k},\omega)$ and $g^{(0)a}_{\pm}(\boldsymbol{k},\omega)$ defined as

$$g^{(0)r}_{\pm}(\boldsymbol{k},\omega) = \frac{1}{\hbar\omega \mp \epsilon_{\boldsymbol{k}} + i\hbar\tau^{(-1)}_{\boldsymbol{k}}}, \quad \text{(A5)}$$

$$g^{(0)a}_{\pm}(\boldsymbol{k},\omega) = (g^{(0)r}_{\pm}(\boldsymbol{k},\omega))^{*}. \quad \text{(A6)}$$

These propagators are related to the magnon propagators (A2) and (A3) through

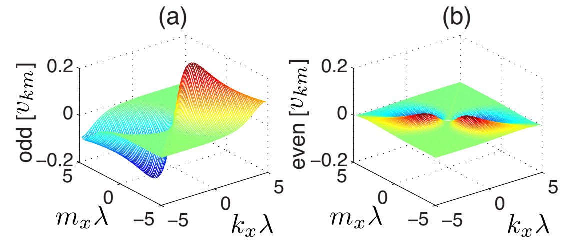

FIG. 12. (Color online) (a) Odd part $(v_{\boldsymbol{km}}-v_{\boldsymbol{mk}})/2$ of coefficient $v_{\boldsymbol{km}}$. (b) Even part $(v_{\boldsymbol{km}}+v_{\boldsymbol{mk}})/2$ of coefficient $v_{\boldsymbol{km}}$.

$$\mathcal{D}^{(0)r} = (I+\sigma_y)g^{(0)r}_{+} - (I-\sigma_y)g^{(0)r}_{-} \quad \text{(A7)}$$

and

$$\mathcal{D}^{(0)a} = (I+\sigma_y)g^{(0)a}_{+} - (I-\sigma_y)g^{(0)a}_{-}. \quad \text{(A8)}$$

The products of the retarded and advanced free magnon propagators, which appear throughout the calculation of the response, can be readily computed by using Eqs. (A8) and (A7) and by noticing that $(I+\sigma_y)(I-\sigma_y)=0$ and $(I\pm\sigma_y)^2=2(I\pm\sigma_y)$.

The scattering potential $\mathcal{V}$ (35) may be divided into a linear term and a "second-order" term with respect to the propagating modes. The linear term does not perturb the correlations of the spin waves since $\langle T\hat{c}^{\dagger}_{\boldsymbol{k}}\hat{c}_{\boldsymbol{k}'}\hat{c}_{\boldsymbol{k}'}\rangle=0$ and $\langle T\hat{c}^{\dagger}_{\boldsymbol{k}}\hat{c}_{\boldsymbol{k}'}\hat{c}^{\dagger}_{\boldsymbol{k}'}\rangle=0$. However the second-order term $-S(u-\dot{x}_0)\Sigma_{\boldsymbol{km}}v_{\boldsymbol{km}}\hat{\theta}_{\boldsymbol{m}}\hat{\phi}_{\boldsymbol{k}}$ modifies these correlations. In the following we assume the coefficient $v_{\boldsymbol{km}}$ to be odd,

$$v_{\boldsymbol{km}} = -v_{\boldsymbol{mk}}. \quad \text{(A9)}$$

Strictly speaking, this assumption does not rigorously hold in the general case $\kappa\neq 0$; however, the plots in Fig. 12 suggest this to be a good approximation.

In the interaction picture, the contour-ordered magnon propagator is given by

$$\mathcal{D}_{\alpha\beta}(\boldsymbol{k},\boldsymbol{k}',\tau-\tau') = -\frac{i}{\hbar}\langle\,|T\exp\left(-\frac{i}{\hbar}\int_C d\tau_1\,\mathcal{V}(\tau_1)\right)\hat{\alpha}_{\boldsymbol{k}}(\tau)\hat{\beta}_{-\boldsymbol{k}'}(\tau')|\,\rangle. \quad \text{(A10)}$$

The linear-response theory focuses on the first-order expansion of $\mathcal{D}_{\alpha\beta}(\boldsymbol{k},\boldsymbol{k}',\tau-\tau')$ with respect to the coupling potential $\mathcal{V}$. We use the superscript (1) to denote the first-order expansion of the magnon propagator. Recalling Wick's theorem and using relationship (A9), we readily obtain

$$\mathcal{D}^{(1)}_{\alpha\beta}(\boldsymbol{k},\boldsymbol{k}',\tau,\tau') = \int_C d\tau_1\,u_{-\boldsymbol{kk}'}(\tau_1)[\mathcal{D}^{(0)}_{\alpha\theta}(\boldsymbol{k},\tau-\tau_1)\times\mathcal{D}^{(0)}_{\phi\beta}(\boldsymbol{k}',\tau_1-\tau') - \mathcal{D}^{(0)}_{\alpha\phi}(\boldsymbol{k},\tau-\tau_1)\times\mathcal{D}^{(0)}_{\theta\beta}(\boldsymbol{k}',\tau_1-\tau')].$$

Employing the 2×2 matrix notation, we finally find

$$\mathcal{D}^{(1)}(\boldsymbol{k},\boldsymbol{k}',\tau,\tau') = \int_C d\tau_1\, iu_{-\boldsymbol{k}\boldsymbol{k}'}(\tau_1) \times \mathcal{D}^{(0)}(\boldsymbol{k},\tau-\tau_1)\sigma_y \mathcal{D}^{(0)}(\boldsymbol{k}',\tau_1-\tau'). \tag{A11, A12}$$

Equation (A12) expresses the linear response of the spin waves to the coupling potential $\mathcal{V}$ (35).

*yann@lemaho-micro.eu
†joo-von.kim@u-psud.fr
‡tatara@phys.metro-u.ac.jp

[1] S. S. P. Parkin, M. Hayashi, and L. Thomas, Science **320**, 190 (2008).
[2] D. A. Allwood, G. Xiong, M. D. Cooke, C. C. Faulkner, D. Atkinson, N. Vernier, and R. P. Cowburn, Science **296**, 2003 (2002).
[3] T. Ono, H. Miyajima, K. Shigeto, K. Mibu, N. Hosoito, and T. Shinjo, Science **284**, 468 (1999).
[4] L. Thomas, M. Hayashi, X. Jiang, R. Moriya, C. Rettner, and S. S. P. Parkin, Nature (London) **443**, 197 (2006).
[5] M. Hayashi, L. Thomas, C. Rettner, R. Moriya, Y. B. Bazaliy, and S. S. P. Parkin, Phys. Rev. Lett. **98**, 037204 (2007).
[6] N. Vernier, D. A. Allwood, D. Atkinson, M. D. Cooke, and R. P. Cowburn, Europhys. Lett. **65**, 526 (2004).
[7] G. Tatara and H. Kohno, Phys. Rev. Lett. **92**, 086601 (2004).
[8] J. Xiao, A. Zangwill, and M. D. Stiles, Phys. Rev. B **73**, 054428 (2006).
[9] A. Vanhaverbeke and M. Viret, Phys. Rev. B **75**, 024411 (2007).
[10] P. E. Falloon, R. A. Jalabert, D. Weinmann, and R. L. Stamps, Phys. Rev. B **70**, 174424 (2004).
[11] X. Waintal and M. Viret, Europhys. Lett. **65**, 427 (2004).
[12] Y. Tserkovnyak, H. J. Skadsem, A. Brataas, and G. E. W. Bauer, Phys. Rev. B **74**, 144405 (2006).
[13] N. L. Schryer and L. R. Walker, J. Appl. Phys. **45**, 5406 (1974).
[14] A. Malozemoff and J. Slonczewski, *Magnetic Domain Walls in Bubble Materials* (American Press, New York, 1979).
[15] L. Berger, J. Appl. Phys. **49**, 2156 (1978).
[16] L. Berger, J. Appl. Phys. **55**, 1954 (1984).
[17] L. Berger, J. Appl. Phys. **71**, 2721 (1992).
[18] A. Thiaville, Y. Nakatani, J. Miltat, and Y. Suzuki, Europhys. Lett. **69**, 990 (2005).
[19] M. D. Stiles, W. M. Saslow, M. J. Donahue, and A. Zangwill, Phys. Rev. B **75**, 214423 (2007).
[20] G. Tatara, H. Kohno, and J. Shibata, Phys. Rep. **468**, 213 (2008).
[21] G. S. D. Beach, C. Knutson, C. Nistor, M. Tsoi, and J. L. Erskine, Phys. Rev. Lett. **97**, 057203 (2006).
[22] R. A. Duine, A. S. Nunez, J. Sinova, and A. H. MacDonald, Phys. Rev. B **75**, 214420 (2007).
[23] S. E. Barnes and S. Maekawa, Phys. Rev. Lett. **95**, 107204 (2005).
[24] H. Kohno, G. Tatara, and J. Shibata, J. Phys. Soc. Jpn. **75**, 113706 (2006).
[25] F. Piechon and A. Thiaville, Phys. Rev. B **75**, 174414 (2007).
[26] J. M. Winter, Phys. Rev. **124**, 452 (1961).
[27] J. F. Janak, Phys. Rev. **134**, A411 (1964).
[28] D. Bouzidi and H. Suhl, Phys. Rev. Lett. **65**, 2587 (1990).
[29] H.-B. Braun, Phys. Rev. B **50**, 16485 (1994).
[30] M. Sparks, *Ferromagnetic Relaxation Theory* (McGraw-Hill, New York, 1964).
[31] D. Ravelosona, D. Lacour, J. A. Katine, B. D. Terris, and C. Chappert, Phys. Rev. Lett. **95**, 117203 (2005).
[32] M. Laufenberg, W. Buhrer, D. Bedau, P.-E. Melchy, M. Klaui, L. Vila, G. Faini, C. A. F. Vaz, J. A. C. Bland, and U. Rudiger, Phys. Rev. Lett. **97**, 046602 (2006).
[33] M. Yamanouchi, D. Chiba, F. Matsukura, T. Dietl, and H. Ohno, Phys. Rev. Lett. **96**, 096601 (2006).
[34] M. Yamanouchi, J. Ieda, F. Matsukura, S. E. Barnes, S. Maekawa, and H. Ohno, Science **317**, 1726 (2007).
[35] A. Yamaguchi, T. Ono, S. Nasu, K. Miyake, K. Mibu, and T. Shinjo, Phys. Rev. Lett. **92**, 077205 (2004).
[36] P. Dagras, M. Kläui, M. Laufenberg, D. Bedau, L. Vila, G. Faini, C. A. F. Vaz, J. A. C. Bland, and U. Ruediger, J. Phys. D **40**, 1247 (2007).
[37] R. Rajaraman, *Solitons and Instantons* (North-Holland Publishing Company, Amsterdam, 1982).
[38] R. M. White, *Quantum Theory of Magnetism* (McGraw-Hill, New York, 1970).
[39] R. E. Prange and V. Korenman, Phys. Rev. B **19**, 4698 (1979).
[40] J. Shibata, G. Tatara, and H. Kohno, Phys. Rev. Lett. **94**, 076601 (2005).
[41] L. Thompson, Masters thesis, University of British Columbia, 2004; arXiv:cond-mat/0510414 (unpublished).
[42] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Pergamon Press, New York, 1981).
[43] A. Thiaville, Y. Nakatani, J. Miltat, and N. Vernier, J. Appl. Phys. **95**, 7049 (2004).
[44] P. Lederer and D. L. Mills, Phys. Rev. **148**, 542 (1966).
[45] V. Vlaminck and M. Bailleul, Science **322**, 410 (2008).
[46] J. Fernández-Rossier, M. Braun, A. S. Núñez, and A. H. MacDonald, Phys. Rev. B **69**, 174412 (2004).
[47] N. V. Prokof'ev and P. C. E. Stamp, Rep. Prog. Phys. **63**, 669 (2000).
[48] A. O. Caldeira and A. J. Leggett, Physica A **121**, 587 (1983).
[49] R. P. Feynman and F. L. Vernon, Ann. Phys. (N.Y.) **24**, 118 (1963).
[50] K. Fujikawa and H. Terashima, Phys. Rev. E **58**, 7063 (1998).
[51] F. Cayssol, D. Ravelosona, C. Chappert, J. Ferré, and J. P. Jamet, Phys. Rev. Lett. **92**, 107202 (2004).
[52] M. Kläui, C. A. F. Vaz, J. A. C. Bland, W. Wernsdorfer, G. Faini, E. Cambril, L. J. Heyderman, F. Nolting, and U. Rüdiger, Phys. Rev. Lett. **94**, 106601 (2005).
[53] A. Thiaville, Y. Nakatani, F. Piechon, J. Miltat, and T. Ono, Eur. Phys. J. B **60**, 15 (2007).